MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

CSC COMPUTATIONAL METHODS IN SYSTEMS AND CONTROL THEORY

20 YEARS 1998–2018

Math Energy

Federal Ministry for Economic Affairs and Energy

Parametric Model Order Reduction for Gas Flow Models

Peter Benner[1], Sara Grundel[1], Christian Himpe[1]

[1]Max Planck Institute for Dynamics of Complex Technical Systems, Computational Methods in Systems and Control Theory

1. Pipe Model

- **Isothermal Euler Equation**:

$$\frac{1}{R_S T}\frac{\partial}{\partial t}\frac{p}{z(p)} = -\frac{1}{S}\frac{\partial}{\partial x}q$$
$$\frac{1}{S}\frac{\partial}{\partial t}q = -\frac{\partial}{\partial x}p \underbrace{-\frac{R_S T}{S^2}\frac{\partial}{\partial x}q^2\frac{z(p)}{p}}_{\text{Inertia Term}} \underbrace{-\frac{g}{R_S T}\frac{p}{z_0(p)}\frac{\partial}{\partial x}h}_{\text{Gravity Term}} \underbrace{-\frac{\lambda R_S T}{2DS^2}\frac{q|q|}{\left(\frac{p}{z(p)}\right)}}_{\text{Friction Term}}$$

  - Nonlinear, hyperbolic, coupled PDE
  - $z(p)$ Compressibility factor (nonlinear)
  - Discard inertia term due to flow speed

- **Variables**:
  - $p(x,t)$ Pressure
  - $q(x,t)$ Mass-flux
- **Constants**:
  - D Pipe diameter
  - $S := \frac{1}{2}D\pi^2$ Cross-section area
  - T Temperature
  - R_S Specific gas constant
  - λ Friction factor (nonlinear)

2. Network Model

- **Graph-Based Approach**:
  - Edges correspond to pipes
  - Nodes correspond to junctions
- **Repetitive modelling**: Four states per pipe:
  - Pressure at inlet $p_i(t)$
  - Mass-flux at inlet $q_i(t)$
  - Pressure at outlet $p_o(t)$
  - Mass-flux at outlet $q_o(t)$
- **Boundary Values**:
  - Supply pressure $s(t)$
  - Demand mass-flux $d(t)$
- **Conservation Laws**:
  1. $p_i^k(t) = s^k(t)$
  2. $\sum q_o^k(t) - \sum q_i^k(t) = d^k(t)$
  - Interconnected PDAE
- **Quantities of Interest**:
  - Demand pressure
  - Supply mass-flux

3. Discretization

- **Spatial**: PDAE $\rightarrow$ DAE
  - Finite differences
  - 1D per pipeline
  - Adaptive refinement
- **Analytic Index Reduction**:
  - Transform DAE to implicit or explicit ODE
  - (tractability) index-1
- **Input-Output System**:
  - Inputs: Boundary values
  - Outputs: Quantities of interest
- **ODE Model**:

$$E\dot{x} = A(\theta)x + Bu + f(x,u,\theta)$$
$$y = Cx$$

- **Center at Steady-State**:

$$E\Delta\dot{x} = A(\theta)(\bar{x}+\Delta x) + Bu + f(\bar{x}+\Delta x, u, \theta)$$
$$y = C(\bar{x}+\Delta x)$$

- **Temporal**:
  - 1st order implicit-explicit
  - Stiff linear part: Implicitly
  - Nonlinear part: Explicitly

4. Parametrization

- **Parameters** ($\theta \subset \mathbb{R}^2$):
  - $\theta_1 := T$ Temperature
  - $\theta_2 := R_S$ Specific gas constant
- **Parametrization**:
  - Non-affine
  - Challenge: Integration rather inefficent
  - Solution: Lump in nonlinear vector field
- **Lumping**:
  - Linear parametric part
  - Compressibility
  - Gravity term
- **Outlook**:
  - Pipe roughness (high-dimensional θ)
  - Combined state and parameter reduction

5. Model Reduction

- **Reducded Order Model**:

$$E_r\dot{x}_r = A_r x_r + B_r u + f_r(x_r, u, \theta)$$
$$\tilde{y} = C_r x_r$$

  such that:

$$\|y - \tilde{y}\| \ll 1$$
$$\dim(x_r) \ll \dim(x)$$

- **Projection-Based Model Reduction**:

$$VEU\dot{x}_r = VAUx_r + VBu + Vf(Ux_r, u, \theta)$$
$$\tilde{y} = CUx_r$$

- **Structured Model Reduction**:

$$U = \begin{pmatrix} U_p & 0 \\ 0 & U_q \end{pmatrix},\ V = \begin{pmatrix} V_p & 0 \\ 0 & V_q \end{pmatrix}$$

- **Parametric Model Order Reduction (pMOR)**:

$$\|y(\theta) - \tilde{y}(\theta)\| \ll 1$$

  - Method I: Stacking
  - Method II: Averaging

6. pMOR Methods

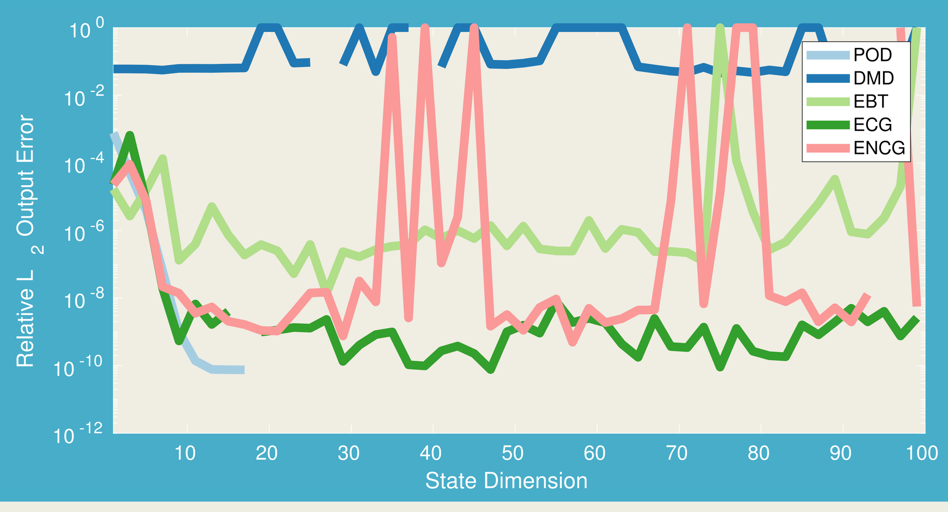
- **Empirical Balanced Truncation**
  1. Empirical controllability Gramian
  2. Empirical observability Gramian
  3. Balancing transformation
- **Empirical Direct Truncation**
  1. Empirical cross Gramian
  2. Balancing transformation
- **Empirical Approximate Truncation**
  1. Empirical nonsymmetric cross Gramian
  2. Balancing transformation
- **Proper Orthogonal Decomposition (POD)**
  1. Empirical controllability Gramian
  2. Principal singular vectors
- **Dynamic Mode Decomposition (DMD)**
  1. Approximate system matrix
  2. Orthogonalized principal eigenvectors

7. Benchmark Model

- **Roadrunner Pipeline**:
  - Pipe length: 329000m
  - Pipe diameter: 0.762m
  - Pipe roughness: 0.0005m
- **Modelling**:
  - Friction factor: Hofer
  - Compressibility factor: Ideal
  - Auto-refine if: $>$ 1000m
- **Network Dimension**:
  - Number of internal nodes: 328
  - Number of supply nodes: 1
  - Number of demand nodes: 1
  - Number of edges: 329
- **System Dimension**:
  - Number of total states: 658
  - Number of pressure-states: 329
  - Number of mass-flux-states: 329
  - Number of inputs: 2
  - Number of outputs: 2

8. Numerical Results

- **Scenario**:
  - Steady supply pressure: 55bar
  - Steady demand flow: 150kg/s
  - Time horizon: 24h
  - Time resolution: 600s
  - Temperature: $[10, 30]^\circ$C
  - Specific gas constant: $[1500, 1550]$J/(kg K)
- **Experiment**:
  - Training: Perturbed steady-state
  - Test: Unsteady scenario
  - Parameter: 10 Uniform samples



Abstract

Planning the dispatch of contracted gas denominations requires various simulations of the involved gas transport infrastructure. Furthermore, due to the growing interplay of traditional gas transport and fluctuating demands related to renewable energies, the number of necessary simulations vastly increases. Mathematically, a system of Euler equations, which are coupled according to the underlying gas network topology, embodies the associated nonlinear and hyperbolic model. Repeated simulation of large networks for varying supply and demand scenarios often necessitates model order reduction. Yet, beyond these variable boundary conditions, further attributes of the network may be uncertain or need to be kept variable throughout simulations, which motivates parametric model order reduction (pMOR).

README

- P. Benner, S. Grundel, C. Himpe, C. Huck, T. Streubel and C. Tischendorf. **Gas Network Benchmark Models**. Submitted, TRR154 Preprint, 2017.
- U. Baur, P. Benner, B. Haasdonk, C. Himpe, I. Martini and M. Ohlberger. **Comparison of Methods for Parametric Model Order Reduction of Time-Dependent Problems**. In: Model Reduction and Approximation: Theory and Algorithms, SIAM: 377–407, 2017.
- A. Alla and J.N. Kutz. **Nonlinear Model Order Reduction via Dynamic Mode Decomposition**. SIAM Journal on Scientific Computing, 39(5), B778–B796, 2017.
- C. Himpe and M. Ohlberger. **A Note on the Cross Gramian for Non-Symmetric Systems**. Systems Science and Control Engineering, 4(1): 199–208, 2016.
- C. Himpe and M. Ohlberger. **Cross-Gramian-Based Combined State and Parameter Reduction for Large-Scale Control Systems**. Mathematical Problems in Engineering, 2014: 1–13, 2014.
- H. Sandberg and R.M. Murray. **Model reduction of interconnected linear systems**. Optimal Control Applications and Methods, 30(3): 225–245, 2009.

Acknowledgement: Supported by the German Federal Ministry for Economic Affairs and Energy, in the joint project: "**MathEnergy** – Mathematical Key Technologies for Evolving Energy Grids", sub-project: Model Order Reduction (Grant number: 0324019**B**).