emgr - Empirical Gramian Framework

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(Software Afternoon)

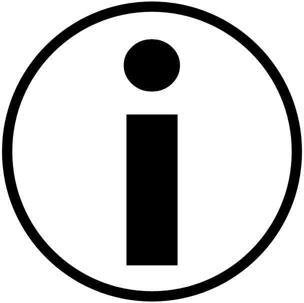
About

General Topic:

- Model Reduction
- System Identification
- Uncertainty Quantification

Outline:

- Mathematical Background
- Capabilities and Features
- Examples and Demos



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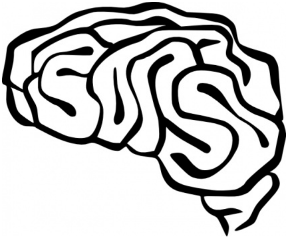
Why?

Tasks:

- fMRI / fNIRS, EEG / MEG Connectivity Analysis
- Hyperbolic Networks
- Discretized PDEs

Challenges:

- Large-Scale ODEs
- Nonlinearites



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emgr (At A Glance)

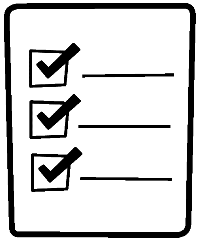
emgr - **EM**pirical **GR**amian Framework

- Version 2.2
- Open-Source (BSD 2-Clause-License)
- Compatible with
  - Octave
  - Matlab
  - (FreeMat)
- about 400 LoC

约

emgr (Areas of Application)

- Model Order Reduction
  - State Reduction
  - Parameter Reduction
  - Combined Reduction
- Parameter Identification
- Sensitivity Analysis



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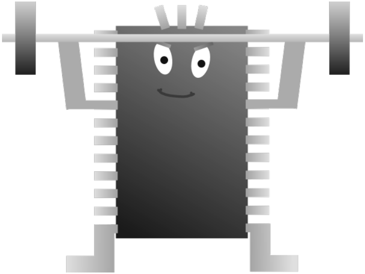
emgr Features

Implementation Details:

- Uniform Interface
- Adaptable Solver Backend
- Only Matrix and Vector Operations
- Generalized Transpositions

Performance Details:

- Vectorized
- Implicit Parallelization
- Explicitly Parallelizable
- Profiled & Benchmarked



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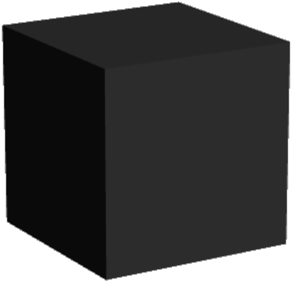
Control Systems

Attributes:

- Input
- (Internal) State
- Output
- Dynamics (ODE / PDE)



General Control Systems

$$
\begin{aligned}
\dot{x}(t) &= f(x(t), u(t)) \\
y(t) &= g(x(t), u(t)) \\
x(0) &= x_0
\end{aligned}
$$

Input / Control:	$u \in \mathbb{R}^M$
State:	$x \in \mathbb{R}^N$
Output:	$y \in \mathbb{R}^O$
Vector Field:	$f : \mathbb{R}^N \times \mathbb{R}^M \times \mathbb{R} \to \mathbb{R}^N$
Output Functional:	$g : \mathbb{R}^N \times \mathbb{R}^M \to \mathbb{R}^O$

Linear Control Systems[1]

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$
$$x(0) = x_0$$

Input / Control:	$u \in \mathbb{R}^M$
State:	$x \in \mathbb{R}^N$
Output:	$y \in \mathbb{R}^O$
System Matrix:	$A \in \mathbb{R}^{N \times N}$
Input Matrix:	$B \in \mathbb{R}^{N \times M}$
Output Matrix:	$C \in \mathbb{R}^{O \times N}$

[1] Adjoint System: $\dot{\hat{x}} = A^{\mathsf{T}} \hat{x} + C^{\mathsf{T}} \hat{u}$; $\hat{y} = B^{\mathsf{T}} \hat{x}$

Large-Scale Systems

Large-Scale Control System:

$$\dot{x} = f(x, u)$$
$$y = g(x, u)$$

State: $x \gg 1$

Input: $u \ll x$

Output: $y \ll x$

Large-Scale Parametrized Control System:

$$\dot{x} = f(x, u, \theta)$$
$$y = g(x, u, \theta)$$

Parameter: $\theta \gg 1$

Controllability & Observability [Moore'81]

Controllability Operator:

$$\mathcal{C}(u) := \int_0^\infty e^{At} B u(t) \mathrm{d}t$$

Observability Operator:

$$\mathcal{O}(x_0) := C e^{At} x_0$$

Controllability Gramian

Controllability Gramian:

$$
\begin{aligned}
W_C &:= \mathcal{C}\mathcal{C}^T \\
&= \int_0^\infty e^{At} B B^T e^{A^T t} \mathrm{d}t \\
&\Rightarrow A W_C + W_C A^T = -B B^T
\end{aligned}
$$

$$W_C$$

Observability Gramian

Observability Gramian:

$$
\begin{aligned}
W_O &:= \mathcal{O}^T \mathcal{O} \\
&= \int_0^\infty e^{A^T t} C^T C e^{At} \mathrm{d}t \\
&\Rightarrow W_O A^T + A W_O = -C^T C
\end{aligned}
$$

$$W_O$$

Cross Gramian [Fernando & Nicholson'83]

Cross Gramian[2] :

$$
\begin{aligned}
W_X &:= \mathcal{C}\mathcal{O} \\
&= \int_0^\infty e^{At} B C e^{At} \mathrm{d}t \\
&\Rightarrow A W_X + W_X A = -BC
\end{aligned}
$$

$$W_X$$

[2] The control system has to be square, better: symmetric!

Model Reduction for Linear Control Systems

Balanced Truncation:

$$W_O \overset{Cholesky}{=} L_O L_O^T$$

$$W_C \overset{Cholesky}{=} L_C L_C^T$$

$$L_C L_O^T \overset{SVD}{=} UDV^T$$

$$\sigma_1 > \ldots > \sigma_N$$

$$\rightarrow U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}, \; V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}^T$$

$$\tilde{A} = V_1 A U_1$$

$$\tilde{B} = V_1 B$$

$$\tilde{C} = C U_1$$

$$\tilde{x}_0 = V_1 x_0$$

Direct Truncation:

$$W_X \overset{SVD}{=} UDV^T$$

$$\sigma_1 > \ldots > \sigma_N$$

$$\rightarrow U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

$$\tilde{A} = U_1^T A U_1$$

$$\tilde{B} = U_1^T B$$

$$\tilde{C} = C U_1$$

$$\tilde{x}_0 = U_1^T x_0$$

Model Reduction for Nonlinear Control Systems

Balanced Truncation:

$$W_O \overset{Cholesky}{=} L_O L_O^T$$
$$W_C \overset{Cholesky}{=} L_C L_C^T$$
$$L_C L_O^T \overset{SVD}{=} UDV^T$$
$$\sigma_1 > \ldots > \sigma_N$$
$$\to U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix},\ V = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}^T$$

$$\tilde{f} = V_1 f(U_1 x, u, \theta)$$
$$\tilde{g} = g(U_1 x, u, \theta)$$
$$\tilde{x}_0 = V_1 x_0$$

Direct Truncation:

$$W_X \overset{SVD}{=} UDV^T$$
$$\sigma_1 > \ldots > \sigma_N$$
$$\to U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

$$\tilde{f} = U_1^T f(U_1 x, u, \theta)$$
$$\tilde{g} = g(U_1 x, u, \theta)$$
$$\tilde{x}_0 = U_1^T x_0$$

Empirical Gramians (About)

System Gramians:

$$W_C = \int_0^\infty (e^{At} B)\,(e^{At} B)^T \mathrm{d}t$$
$$W_O = \int_0^\infty (e^{A^T t} C^T)\,(e^{A^T t} C^T)^T \mathrm{d}t$$
$$W_X = \int_0^\infty (e^{At} B)\,(e^{A^T t} C^T)^T \mathrm{d}t$$

Integrands are just impulse responses!

$\rightarrow$ Compute gramians empirically.

emgr (Capabilities)

Empirical Gramians:

1. Empirical Controllability Gramian (W_C)
2. Empirical Observability Gramian (W_O)
3. Empirical Linear Cross Gramian (W_Y)
4. Empirical Cross Gramians (W_X)

5. Empirical Sensitivity Gramian (W_S)
6. Empirical Identifiability Gramian (W_I)
7. Empirical Joint Gramian (W_J)

emgr (Signature)

Minimal Signature:

`W` = emgr(f, g, q, t, w);

Full Signature:

`W` = emgr(f, g, q, t, w, pr, nf, ut, us, xs, um, xm, yd);

- f - System Function Handle
- g - Output Function Handle
- q - System Dimension (Inputs, States, Outputs)
- t - Time (Start, Step, Stop)
- w - Gramian Type ('c', 'o', 'x', 'y', 's', 'i', 'j')
- pr - Parameters
- nf - Configuration
- ut - Input
- us - Steady State Input
- xs - Steady State
- um - Input Scales
- xm - State Scales
- yd - Data

Empirical Gramians (Perturbations)

Impulse Input Perturbations: $u^{hij}(t) = c_h S_i e_j \delta(t)$

$$E_u = \{e_i \in \mathbb{R}^m; \|e_i\| = 1; e_i e_{j \neq i} = 0; i = 1, \dots, m\},$$
$$R_u = \{S_i \in \mathbb{R}^{m \times m}; S_i^T S_i = \mathbb{1}; i = 1, \dots, s\},$$
$$Q_u = \{c_i \in \mathbb{R}; c_i > 0; i = 1, \dots, q\},$$

Initial State Perturbation: $x_0^{kla} = d_k T_l f_a$

$$E_x = \{f_i \in \mathbb{R}^n; \|f_i\| = 1; f_i f_{j \neq i} = 0; i = 1, \dots, n\},$$
$$R_x = \{T_i \in \mathbb{R}^{n \times n}; T_i^T T_i = \mathbb{1}; i = 1, \dots, t\},$$
$$Q_x = \{d_i \in \mathbb{R}; d_i > 0; i = 1, \dots, r\}.$$

emgr (Perturbations)

Impulse Input Perturbations: $u^{hij}(t) = c_h S_i e_j \delta(t)$

$$
\begin{aligned}
E_u &= \{\mathbf{e}_{i=1\dots m}^{\mathbb{R}^m}\}, \\
R_u &= \{-\mathbb{1}, \mathbb{1}\}, \\
Q_u &= \{c_i \in \mathbb{R}; c_i > 0; i = 1, \dots, q\},
\end{aligned}
$$

Initial State Perturbation: $x_0^{kla} = d_k T_l f_a$

$$
\begin{aligned}
E_x &= \{\mathbf{e}_{i=1\dots n}^{\mathbb{R}^n}\}, \\
R_x &= \{-\mathbb{1}, \mathbb{1}\}, \\
Q_x &= \{d_i \in \mathbb{R}; d_i > 0; i = 1, \dots, r\}.
\end{aligned}
$$

Example System[3]

Linear MIMO Control System:

$$
\begin{aligned}
u(t) &= \delta(t) \\
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) \\
x_0 &= 0
\end{aligned}
$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$

Attributes:

- State-Space Symmetric:

  $A = A^T, \; B = C^T$

$\Rightarrow \; W_C = W_O = W_X$

[3] Random System via Inverse Lypunov Procedure (see [Smith'03])

Empirical Controllability Gramian [Lall et al'99]

Analytic Controllability Gramian

$$W_C = \int_0^\infty e^{At} BB^T e^{A^T t} \mathrm{d}t$$

Linear Empirical Controllability Gramian

$$W_C' = \int_0^\infty x(t)\, x^T(t) \mathrm{d}t$$

Empirical Controllability Gramian

$$W_C'' = \frac{1}{|Q_u||R_u|} \sum_{h=1}^{|Q_u|} \sum_{i=1}^{|R_u|} \sum_{j=1}^{m} \frac{1}{c_h^2} \int_0^\infty \Psi^{hij}(t) \mathrm{d}t,$$

$$\Psi^{hij}(t) = (x^{hij}(t) - \bar{x})(x^{hij}(t) - \bar{x})^T \in \mathbb{R}^{n \times n}$$

Discrete Empirical Controllability Gramian

$$W_C''' = \frac{1}{|Q_u||R_u|} \sum_{h=1}^{|Q_u|} \sum_{i=1}^{|R_u|} \sum_{j=1}^{m} \frac{\Delta t}{c_h^2} \sum_{t=0}^{\mathfrak{T}} \Psi_t^{hij},$$

$$\Psi_t^{hij} = (x_t^{hij} - \bar{x})(x_t^{hij} - \bar{x})^T \in \mathbb{R}^{n \times n}$$

Example: POD

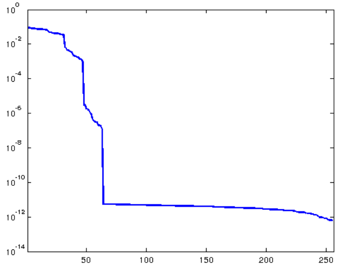
System:

$$\dot{x} = Ax + Bu$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$

Relative L2-Error:



Code:

```
f = @(x,u,p) A*x+B*u;
g = 1;
q = [16,256,16];
t = [0,0.01,1.0];
WC = emgr(f,g,q,t,'c');
[U, D, V] = svd(WC);
```

Empirical Observability Gramian [Lall et al'99]

Analytic Observability Gramian

$$W_O = \int_0^\infty e^{A^T t} C^T C e^{At} \mathrm{d}t$$

Linear Empirical Observability Gramian

$$W'_O = \int_0^\infty \hat{x}(t)\, \hat{x}^T(t) \mathrm{d}t$$

Empirical Observability Gramian

$$W''_O = \frac{1}{|Q_x||R_x|} \sum_{k=1}^{|Q_x|} \sum_{l=1}^{|R_x|} \frac{1}{d_k^2} \int_0^\infty T_l \Psi^{kl}(t) T_l^T \mathrm{d}t,$$

$$\Psi^{kl}_{ab} = (y^{kla}(t) - \bar{y})^T (y^{klb}(t) - \bar{y}) \in \mathbb{R}$$

Discrete Empirical Observability Gramian

$$W'''_O = \frac{1}{|Q_x||R_x|} \sum_{k=1}^{|Q_x|} \sum_{l=1}^{|R_x|} \frac{\Delta t}{d_k^2} \sum_{t=0}^{\mathfrak{T}} T_l \Psi_t kl T_l^T,$$

$$\Psi^{kl}_{t,ab} = (y_t^{kla} - \bar{y})^T (y_t^{klb} - \bar{y}) \in \mathbb{R}$$

Example: Balanced Truncation

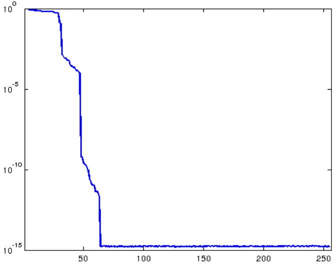
System:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$

Relative L2-Error:



Code:

```
f = @(x,u,p) A*x+B*u;
g = @(x,u,p) C*x;
q = [16,256,16];
t = [0,0.01,1.0];
WC = emgr(f,g,q,t,'c');
WO = emgr(f,g,q,t,'o');
[U, D, V] = balance(WC,WO);
```

Example: Balanced Truncation

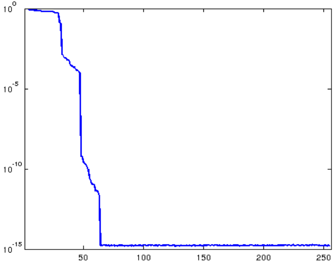
System:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$

Relative L2-Error:



Code:

```
f = @(x,u,p) A*x+B*u;
F = @(x,u,p) A'*x+C'*u;
q = [16,256,16];
t = [0,0.01,1.0];
WC = emgr(f,1,q,t,'c');
WO = emgr(F,1,q,t,'c');
[U, D, V] = balance(WC,WO);
```

Empirical Cross Gramian [Streif'06, H.'14]

Analytic Cross Gramian

$$W_X = \int_0^\infty e^{At} B C e^{At} \mathrm{d}t$$

Linear Empirical Cross Gramian

$$W_X' = \int_0^T x(t) \hat{x}^T(t) \mathrm{d}t$$

Example: (Linear) Direct Truncation

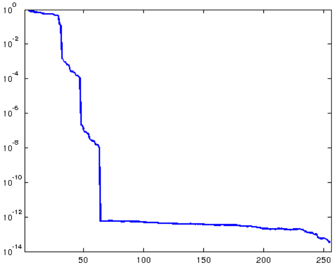
System:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$

Relative L2-Error:



Code:

```
f = @(x,u,p) A*x+B*u;
g = @(x,u,p) A'*x+C'*u;
q = [16,256,16];
t = [0,0.01,1.0];
WY = emgr(f,g,q,t,'y');
[U, D, V] = svd(WY);
```

Empirical Cross Gramian [Streif'06, H.'14]

Analytic Cross Gramian

$$W_X = \int_0^\infty e^{At} B C e^{At} \mathrm{d}t$$

Linear Empirical Cross Gramian

$$W_X' = \int_0^T x(t) \hat{x}^T(t) \mathrm{d}t$$

Empirical Cross Gramian

$$W_X'' = \frac{1}{|Q_u||R_u|m|Q_x||R_x|} \sum_{h=1}^{|Q_{\boldsymbol{u}}|} \sum_{i=1}^{|R_{\boldsymbol{u}}|} \sum_{j=1}^{m} \sum_{k=1}^{|Q_{\boldsymbol{x}}|} \sum_{l=1}^{|R_{\boldsymbol{x}}|} \frac{1}{c_h d_k} \int_0^\infty T_l \Psi^{hijkl}(t) T_l^T \mathrm{d}t,$$

$$\Psi_{ab}^{hijkl}(t) = f_b^T T_k^T (x^{hij}(t) - \bar{x}) e_i^T S_h^T (y^{kla}(t) - \bar{y})$$

Discrete Empirical Cross Gramian

$$W_X''' = \frac{1}{|Q_u||R_u|m|Q_x||R_x|} \sum_{h=1}^{|Q_{\boldsymbol{u}}|} \sum_{i=1}^{|R_{\boldsymbol{u}}|} \sum_{j=1}^{m} \sum_{k=1}^{|Q_{\boldsymbol{x}}|} \sum_{l=1}^{|R_{\boldsymbol{x}}|} \frac{\Delta t}{c_h d_k} \sum_{t=0}^{\mathfrak{T}} T_l \Psi_t^{hijkl} T_l^T,$$

$$\Psi_{t,ab}^{hijkl} = f_b^T T_k^T (x_t^{hij} - \bar{x}) e_i^T S_h^T (y_t^{kla} - \bar{y})$$

Example: Direct Truncation

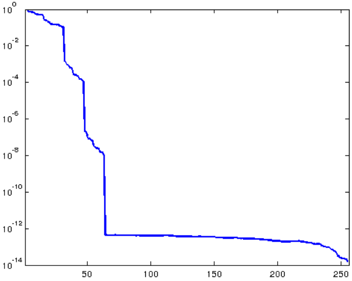
System:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$

Relative L2-Error:



Code:

```
f = @(x,u,p) A*x+B*u;
g = @(x,u,p) C*x;
q = [16,256,16];
t = [0,0.01,1.0];
WX = emgr(f,g,q,t,'x');
[U, D, V] = svd(WX);
```

Parametrized Example Systems[4]

Parametrized General Control System:

$$\dot{x}(t) = f(x(t), u(t), \theta)$$
$$y(t) = g(x(t), u(t), \theta)$$

Parametrized Linear Control System:

$$\dot{x}(t) = A(\theta)x(t) + B(\theta)u(t)$$
$$y(t) = C(\theta)x(t)$$

---

[4] Diagonal Entries of System Matrix A are considered parameters here.

Empirical Sensitivity Gramian [Sun'06, H.'13]

Parameter Decomposed System:

$$\dot{x} = f(x, u) + \sum_{k=1}^{P} f(x, \theta_k)$$

$$\Rightarrow W_C = W_{C,0} + \sum_{k=1}^{P} W_{C,k}$$

Sensitivity Gramian:

$$W_{S,ii} = \mathrm{tr}(W_{C,i})$$

Example: Sensitivity Analysis

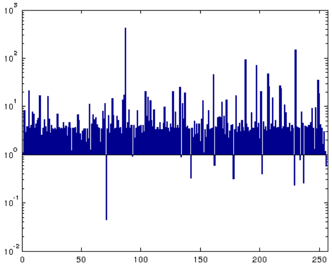
System:

$$\dot{x} = A(\theta)x + Bu$$
$$y = Cx$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$
- Parameter: $\theta \in \mathbb{R}^{256}$

Parameter Sensitivity:



Code:

```
f = @(x,u,p) (A+diag(p))*x+B*u;
g = @(x,u,p) C*x;
q = [16,256,16];
t = [0,0.01,1.0];
WS = emgr(f,g,q,t,'s',p,0,1,1);
```

Empirical Identifiability Gramian [Geffen'08]

Parameter Augmented System:

$$\begin{pmatrix} \dot{x} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} C & 0 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix}$$

$$\begin{pmatrix} x(0) \\ \theta(0) \end{pmatrix} = \begin{pmatrix} x_0 \\ \theta_0 \end{pmatrix}$$

Observability Gramian of Parameter Augmented System:

$$\widetilde{W_O} = \begin{pmatrix} W_O & W_M \\ W_M^T & W_P \end{pmatrix}$$

Identifiability Gramian:
(Schur Complement of Augmented Observability Gramian)

$$W_I := W_P - W_M W_O^{-1} W_M^T \approx W_P$$

Example: Parameter Reduction

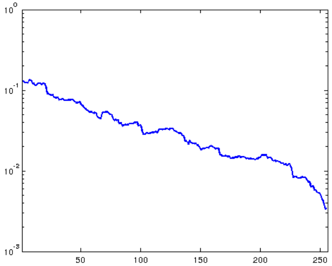
System:

$$\dot{x} = A(\theta)x + Bu$$
$$y = Cx$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$
- Parameter: $\theta \in \mathbb{R}^{256}$

Relative L2-Error:



Code:

```
f = @(x,u,p) (A+diag(p))*x+B*u;
g = @(x,u,p) C*x;
q = [16,256,16];
t = [0,0.01,1.0];
WI = emgr(f,g,q,t,'i',p,0,1,0,1);
[P, S, Q] = svd(WI{2});
```

Empirical Joint Gramian [H.'14]

Parameter Augmented System:

$$\begin{pmatrix} \dot{x} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u$$

$$y = \begin{pmatrix} C & 0 \end{pmatrix} \begin{pmatrix} x \\ \theta \end{pmatrix}$$

$$\begin{pmatrix} x(0) \\ \theta(0) \end{pmatrix} = \begin{pmatrix} x_0 \\ \theta_0 \end{pmatrix}$$

Joint Gramian:
(Cross Gramian of Parameter Augmented System)

$$W_J = \begin{pmatrix} W_X & W_M \\ 0 & 0 \end{pmatrix}$$

Cross-Identifiability Gramian:
(Schur Complement of Symmetric Part of Joint Gramian)

$$W_{\tilde{I}} := -\frac{1}{2} W_M (W_X + W_X^T)^{-1} W_M^T$$

Example: Combined Reduction

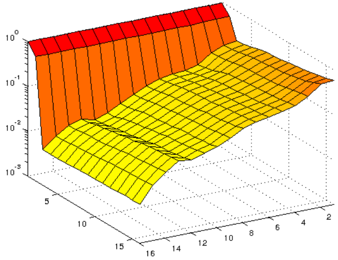
System:

$$\dot{x} = A(\theta)x + Bu$$
$$y = Cx$$

Dimensions:

- Input: $u(t) \in \mathbb{R}^{16}$
- State: $x(t) \in \mathbb{R}^{256}$
- Output: $y(t) \in \mathbb{R}^{16}$
- Parameter: $\theta \in \mathbb{R}^{256}$

Relative L2-Error:



Code:

```
f = @(x,u,p) (A+diag(p))*x+B*u;
g = @(x,u,p) C*x;
q = [16,256,16];
t = [0,0.01,1.0];
WJ = emgr(f,g,q,t,'j',p,0,1,0,1);
[U, D, V] = svd(WJ{1});
[P, S, Q] = svd(WJ{2});
```

More **emgr** Use Cases

- Sensitivity Analysis via System Gain
- Nonlinearity Quantification
- Decentralized Control
- Solve Lyapunov Equations and Sylvester Equations

emgr (Signature)

Minimal Signature:

`W` = emgr(f, g, q, t, w);

Full Signature:

`W` = emgr(f, g, q, t, w, pr, nf, ut, us, xs, um, xm, yd);

- f - System Function Handle
- g - Output Function Handle
- q - System Dimension
  (Inputs, States, Outputs)
- t - Time
  (Start, Step, Stop)
- w - Gramian Type
  ('c', 'o', 'x', 'y', 's', 'i', 'j')

- pr - Parameters
- nf - Configuration
- ut - Input
- us - Steady State Input
- xs - Steady State
- um - Input Scales
- xm - State Scales
- yd - Data

emgr (Configuration Options)

1. **Center**: Steady-State, Mean, Median, Last, Principal, Zero
2. **Input Scale Sequence**: Linear, Log, Geometric, Single
3. **State Scale Sequence**: Linear, Log, Geometric, Single
4. **Input Rotations**: Unit, Reciproce, Dyadic, Single
5. **State Rotations**: Unit, Reciproce, Dyadic, Single
6. **Double Run**?
7. **Use Measured Data**?
8. **Robust Parameters**: None, Active or Passive?
9. **Adjust Parameter Scales**?
10. **Schur Complement on** W_I?
11. **Enforce Gramian Symmetry**?
12. **Solver/Integrator**: RK1,AB2,RRK2,ARK3,Leapfrog,CUSTOM

    Solver signature: `x = CUSTOM_ODE(f,h,T,z,u,p)`

Public Service Announcement

- Download
- Documentation
- Demos

source: clker.com

at:

http://gramian.de

Outlook

Technical:

- Symmetrizers
- Non-Symmetric Cross Gramians
- Parametrized Linear Cross Gramian

Implementation:

- Accelerator Offloading (Xeon Phi)
- Python Version
  (into pyMOR: `http://pymor.org`)
- (Julia Version)
- (asm.js Version)

Listings

- ORMS (Oberwolfach References on Mathematical Software)
- swMATH (Information Service for Mathematical Software)
- MoRePas (Model Reduction for Parametrized Systems)
- MORwiki (Model Order Reduction Wiki)
- GitHub

tl;dl

emgr - Empirical Gramian Framework

http://gramian.de

Thanks!

Code & References

- Companion Source Code: `http://j.mp/esco2014`

C. Himpe, M. Ohlberger; "**A Unified Software Framework for Empirical Gramians**"; Journal of Mathematics 2013:1-6, 2013

C. Himpe, M. Ohlberger; "**Cross-Gramian Based Combined State and Parameter Reduction for Large-Scale Control Systems**"; Mathematical Problems in Engineering (Accepted), 2014

C. Himpe, M. Ohlberger; "**Model Reduction for Complex Hyperbolic Networks**" Proceedings of the ECC'14 (Accepted), 2014