MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC

COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

If the Mountain Won't Come ...
... Reduced Order Transport

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The Transporter: Reduced

What is transport aka advection?

- Local effect.
- Propagation over time.

What's the problem with MOR for transport?

- State-space trajectory has no low-rank.
- Slow decay of singular values.

A Trip to Hyperboli-city

Hyperbolic Symptoms:

- Jacobian has real eigenvalues
- and is diagonalizable.

Numerical Medicine:

- CFL (Courant-Friedrichs-Lewy) condition
- SSP (Strong-Stability-Preserving) integrators

Hyper, Hyper

Hyperbolc PDE Examples:

- Advection Equation (1st Order Linear):
$$\frac{\partial u}{\partial t} = a \frac{\partial u}{\partial x}$$
- Wave Equation (2nd Order Linear):
$$\frac{\partial^2 u}{\partial t^2} = a \frac{\partial^2 u}{\partial x^2}$$
- (Inviscid) Burgers' Equation (Quasi-Linear):
$$\frac{\partial u}{\partial t} = u \frac{\partial u}{\partial x}$$
- (Isothermal) Euler Equations (Nonlinear):
$$\begin{aligned} \frac{\partial \rho}{\partial t} &= -\frac{\partial q}{\partial x} \\ \frac{\partial q}{\partial t} &= -a^2 \frac{\partial \rho}{\partial x} + b \frac{q|q|}{\rho} - g\rho \frac{\partial h}{\partial x} \end{aligned}$$

Wave Centering[1]

1. Define center C: $\int_0^C [f(x)]^2 \,\mathrm{d}\, t = \int_C^N [f(x)]^2 \,\mathrm{d}\, t$.
2. Determine center for each time step.
3. Center trajectory around C.

- Computationally easy.
- Theoretically low ranks possible.
- Practically difficult.

[1] S. Glavaski, J.E. Marsden, and R.M. Murray. **Model Reduction, Centering, and the Karhunen-Loeve Expansion**. Proceedings of the 37th IEEE Conference on Decision & Control: 2071–2076, 1998.

Stability-Preserving Inner Products[2]

1. Linearize Model.
2. Determine inner product P for **some** $Q > 0$: $A^\intercal P + PA = -Q$.
3. Build linear reduced model:
   $A_r := U^\intercal PAU$, $B_r := U^\intercal PB$, $C := CU$.

- Computationally OK.
- Theoretically doable (works for linearized Euler equations).
- Practically what Q?

[2] I. Kalashnikova, M.F. Barone, S. Arunajatesan and B.G. van Bloemen Waanders. **Construction of energy-stable projection-based reduced order models**. Applied Mathematics and Computation 249: 569–596, 2014.

Reproducing Kernel Hilbert Spaces[3]

1. Compute projections using a kernel method.
2. Mind the gap: Don't forget to center.
3. Construct ROM: Map to original space (Ay, there is the rub).

- Computationally challenging (Solve large optimization problem).
- Theoretically supported.
- Practically maybe doable.

[3] J. Bouvrie and B. Hamzi. **Kernel Methods for the Approximation of Nonlinear Systems**. arXiv math.OC(1108.2903): 1–31, 2016.

"It could be worse."[4]

Gas Transport Networks in MathEnergy:

- Large-Scale (10^4, Many-Query)
- Hyperbolic (Conservation Laws)
- Nonlinear (Possibly Non-Differentiable)
- Descriptor System (Better Decoupling)

[4] Leia Organa after jumping down a garbage chute into a trash compactor on the death star.

All implemented model reduction techniques (#datadriven):

- Unstructured / Structured Empirical Cross Gramian
- Unstructured / Structured Empirical Balanced Truncation
- Unstructured / Structured Proper Orthogonal Decomposition
- Unstructured / Structured Dynamic Mode Decomposition

can all handle the previous hyperbolicity treatments.

[5] Raoul Duke on the edge of the desert when the drugs began to take hold.

That's All Folks!

Questions:

- Any war stories about MOR for hyperbolic systems?
- Enough redundancy in the networks to not care?
- Opinions or further methods?

P.S.: Other methods considered: Freezing, Shifted POD