


## Gas Network Model Reduction:

- Generic Model Reduction
- Linear Model Reduction
- Affine Model Reduction
- Structured Model Reduction
- Parametric Model Reduction
- Combined Reduction
- Hyperreduction


in the **MathEnergy** Project.



### Complexity Reduction in Gas Networks via Model Reduction

Peter Benner<sup>1</sup>, Sara Grunzel<sup>1</sup>, Christian Himpe<sup>1</sup>

<sup>1</sup>Max-Planck-Institut für Dynamische Komplexe Systeme, Computational Methods in Systems and Control, Tübingen

Abstract	Gas Network Model	A Step-By-Step Guide
<p>The growing interest of renewable energy requires a change in management of gas networks, as supply and demand become increasingly volatile. To reduce the operation of the gas network, many scenario simulations of a large-scale model are conducted prior to the dispatch. Model reduction alleviates the associated computational complexity by providing surrogate models with asymptotic behavior.</p>	<p>Symbolic Operator Index-Reduced Nonlinear State Equations</p> $\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -A & B \\ 0 & -C \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} d \\ 0 \end{pmatrix} + \begin{pmatrix} \sigma \\ 0 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix}$ <ul style="list-style-type: none"> <li>• Parameters: <math>n \times n, m \times n, n \times 1</math></li> <li>• State: <math>n \times 1</math></li> <li>• Input: <math>m \times 1</math></li> <li>• Output: <math>n \times 1</math></li> <li>• Steady-State: <math>x_{ss} = -A^{-1}Bd</math></li> <li>• Transfer Function: <math>G(s) = C(sI - A)^{-1}B + D</math></li> </ul>	<p><b>Outline:</b></p> <ol style="list-style-type: none"> <li>1. Generic Model Reduction</li> <li>2. Linear Model Reduction</li> <li>3. Affine Model Reduction</li> <li>4. Structured Model Reduction</li> <li>5. Parametric Model Reduction</li> <li>6. Combined Reduction</li> <li>7. Hyper Reduction</li> </ol>
1a. Generic Model	1b. Generic Model Reduction	1c. Projection-Based Model Reduction
<p><b>Problem Statement:</b> Non-Output System</p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• State: <math>n \times 1</math></li> <li>• Input: <math>m \times 1</math></li> <li>• Output: <math>p \times 1</math></li> <li>• Dimensional: <math>n, m, p \times n, m \times n, n \times p</math></li> </ul>	<p><b>Reduced State Model:</b></p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• Reduced State: <math>n \times 1</math></li> <li>• Reduced Input: <math>m \times 1</math></li> <li>• Reduced Output: <math>p \times 1</math></li> <li>• Reduced Dimension: <math>n, m, p \times n, m \times n, n \times p</math></li> </ul>	<p><b>Low Dimensional Projection Input-Output System:</b></p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• Reduced Transfer Function: <math>G(s) = C(sI - A)^{-1}B + D</math></li> <li>• Dimensionality: <math>n, m, p \times n, m \times n, n \times p</math></li> <li>• Reduced State: <math>n \times 1</math></li> <li>• Reduced Input: <math>m \times 1</math></li> <li>• Reduced Output: <math>p \times 1</math></li> </ul>
2. Linear Model Reduction	3. Affine Model Reduction	4. Structured Model Reduction
<p><b>Reduced State Model:</b></p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• Parameters can be updated online</li> <li>• Reduced State: <math>n \times 1</math></li> <li>• Reduced Input: <math>m \times 1</math></li> <li>• Reduced Output: <math>p \times 1</math></li> <li>• Reduced Dimension: <math>n, m, p \times n, m \times n, n \times p</math></li> </ul>	<p><b>Affine Reduced Input-Output System:</b></p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• Steady State: <math>x_{ss} = -A^{-1}Bd</math></li> <li>• Reduced State: <math>n \times 1</math></li> <li>• Reduced Input: <math>m \times 1</math></li> <li>• Reduced Output: <math>p \times 1</math></li> <li>• Transfer Function: <math>G(s) = C(sI - A)^{-1}B + D</math></li> </ul>	<p><b>Structured Reduced State Model:</b></p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• Reduced State: <math>n \times 1</math></li> <li>• Reduced Input: <math>m \times 1</math></li> <li>• Reduced Output: <math>p \times 1</math></li> <li>• Reduced Dimension: <math>n, m, p \times n, m \times n, n \times p</math></li> </ul>
5. Parametric Model Reduction	6. Combined Reduction*	7. Hyper Reduction*
<p><b>Parameter Input-Output System:</b></p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• Parameters: <math>n \times n, m \times n, n \times 1</math></li> <li>• State: <math>n \times 1</math></li> <li>• Input: <math>m \times 1</math></li> <li>• Output: <math>p \times 1</math></li> <li>• Parameters Dimensional: <math>n, m, p \times n, m \times n, n \times p</math></li> </ul>	<p><b>Combined State and Parameter Reduction:</b></p> $\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \\ z &= Gx + Hx \end{aligned}$ <ul style="list-style-type: none"> <li>• Reduced Transfer Function: <math>G(s) = C(sI - A)^{-1}B + D</math></li> <li>• Dimensionality: <math>n, m, p \times n, m \times n, n \times p</math></li> <li>• Reduced State: <math>n \times 1</math></li> <li>• Reduced Input: <math>m \times 1</math></li> <li>• Reduced Output: <math>p \times 1</math></li> </ul>	<p><b>Listing Methods:</b></p> <ol style="list-style-type: none"> <li>1. <math>\mathcal{H}_2</math> Norm</li> <li>2. <math>\mathcal{H}_\infty</math> Norm</li> <li>3. <math>\mathcal{H}_2</math> Norm</li> <li>4. <math>\mathcal{H}_\infty</math> Norm</li> <li>5. <math>\mathcal{H}_2</math> Norm</li> <li>6. <math>\mathcal{H}_\infty</math> Norm</li> <li>7. <math>\mathcal{H}_2</math> Norm</li> <li>8. <math>\mathcal{H}_\infty</math> Norm</li> </ol>
Reduced Gas Network Model	Projection Computation	Furthermore ...
<p><b>Feeding it all together:</b></p> $\begin{pmatrix} \dot{x} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -A & B \\ 0 & -C \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} d \\ 0 \end{pmatrix} + \begin{pmatrix} \sigma \\ 0 \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix}$ <ul style="list-style-type: none"> <li>• Reduced Transfer: <math>G(s) = C(sI - A)^{-1}B + D</math></li> <li>• Reduced State: <math>n \times 1</math></li> <li>• Reduced Input: <math>m \times 1</math></li> <li>• Reduced Output: <math>p \times 1</math></li> <li>• Reduced Dimension: <math>n, m, p \times n, m \times n, n \times p</math></li> </ul>	<p><b>Nonlinear Data-Driven Models Considered:</b></p> <ul style="list-style-type: none"> <li>• Empirical Balanced Truncation</li> <li>• Empirical Cross Gramian</li> <li>• Empirical Non-Symmetric Cross Gramian</li> <li>• Proper Orthogonal Decomposition</li> <li>• Dynamic Mode Decomposition</li> </ul>	<p><b>Model reduction methods have different properties, i.e.:</b></p> <ul style="list-style-type: none"> <li>• Stability: <math>\mathcal{H}_2</math> vs. <math>\mathcal{H}_\infty</math> vs. <math>\mathcal{H}_\infty</math> vs. <math>\mathcal{H}_2</math></li> <li>• Accuracy: <math>\mathcal{H}_2</math> vs. <math>\mathcal{H}_\infty</math> vs. <math>\mathcal{H}_2</math> vs. <math>\mathcal{H}_\infty</math></li> <li>• Speed: <math>\mathcal{H}_2</math> vs. <math>\mathcal{H}_\infty</math> vs. <math>\mathcal{H}_2</math> vs. <math>\mathcal{H}_\infty</math></li> <li>• Memory: <math>\mathcal{H}_2</math> vs. <math>\mathcal{H}_\infty</math> vs. <math>\mathcal{H}_2</math> vs. <math>\mathcal{H}_\infty</math></li> </ul> <p>Get this paper: </p>
BIBLIOGRAPHY		
<p>Benner, P., Grunzel, S., Himpe, C. (2022) Complexity Reduction of Gas Networks via Model Reduction. In: Proceedings of the 2022 American Nuclear Society Meeting on Nuclear Energy for the Sustainable Future, 2022, pp. 1-12.</p> <p>Benner, P., Grunzel, S., Himpe, C. (2022) Complexity Reduction of Gas Networks via Model Reduction. In: Proceedings of the 2022 American Nuclear Society Meeting on Nuclear Energy for the Sustainable Future, 2022, pp. 1-12.</p> <p>Benner, P., Grunzel, S., Himpe, C. (2022) Complexity Reduction of Gas Networks via Model Reduction. In: Proceedings of the 2022 American Nuclear Society Meeting on Nuclear Energy for the Sustainable Future, 2022, pp. 1-12.</p> <p>Benner, P., Grunzel, S., Himpe, C. (2022) Complexity Reduction of Gas Networks via Model Reduction. In: Proceedings of the 2022 American Nuclear Society Meeting on Nuclear Energy for the Sustainable Future, 2022, pp. 1-12.</p> <p>Benner, P., Grunzel, S., Himpe, C. (2022) Complexity Reduction of Gas Networks via Model Reduction. In: Proceedings of the 2022 American Nuclear Society Meeting on Nuclear Energy for the Sustainable Future, 2022, pp. 1-12.</p>		