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CSC

COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

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# Model Reduction for Gas Networks: The Good, the Bad, the Ugly

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# Intro: Semi-Discrete Model

**Input-Output System** (PDAE after index reduction and spatial discretization):

$$\begin{pmatrix} E_p & 0 \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & A_{pq}(\theta) \\ A_{qp}(\theta) & 0 \end{pmatrix} \begin{pmatrix} p \\ z^2(p,\theta)q \end{pmatrix} + \begin{pmatrix} 0 & B_d \\ B_s & 0 \end{pmatrix} \begin{pmatrix} p_s \\ q_d \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p,q,\theta) \end{pmatrix}$$

$$\begin{pmatrix} p_d \\ q_s \end{pmatrix} = \begin{pmatrix} C_d & 0 \\ 0 & C_s \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

**Input**:

- Pressure @ Supply $p_s$
- Mass-flow @ Demand $q_d$

**State**:

- Pressure $p$
- Mass-flow $q$

**Output**:

- Pressure @ Demand $p_d$
- Mass-flow @ Supply $q_s$

# Good: System Structure

**Exploitable Structure**:

| | |
|---|---|
| System: | Input-output form |
| Stability: | Locally exponentially stable |
| Structure: | Pressure and mass-flow states |
| Square: | Same number of inputs and outputs |
| Symmetry: | Linearization is Port-Hamiltonian |
| Control: | Control-affine system |
| Locality: | Single steady-state |
| Split: | Linear (stiff) and Nonlinear parts |
| Parameter: | Low-dimensional, but non-affine |

# Good: Friction Nonlinearity

**Friction Term**:

$$f_q(p, q, \theta) = -z(p, \theta) \frac{\lambda(q)}{C} \frac{q|q|}{p}$$

**A Well-Mannered Nonlinearity**:

Locality: Element-wise

Stability: Locally stable

Friction Factor: Static

Smoothness: Smooth enough

Nonlinearity: Quadratic, but not quadratic

# Bad: Non-Pipes

**Non-Pipe Elements**:

Short Pipes, Resistors, Compressors, Coolers, Regulators, Heaters

**Issues and Treatments** (everything is a pipe):

| | |
|---|---|
| Short Pipe: | Pipe without friction! |
| Resistor: | Pipe with super-friction? |
| Compressors: | Pipe with anti-fricton? |
| Coolers: | Compressor super-model? |
| Regulators: | Anti-compressor? |
| Heaters: | Regulator super-model? |

# Bad: Multi-Scales

**Units and Scales**:

$$[p] = \mathrm{Pa}$$
$$[q] = \frac{\mathrm{kg}}{\mathrm{s}}$$
$$\frac{\mathcal{O}(p)}{\mathcal{O}(q)} = 10^5$$

**Issues and Treatments**:

Structured: Model reduction for interconnected systems!

Internal: Rescale pressure model to [bar]?

External: Scale pressure output to [bar]?

Gramians: Weight via inner product?

Comparison: Weighted norms?

# Ugly: Compressibility Nonlinearity

**Compressibility Factor** (Truncated Virial Expansion):

$$z(p, \theta) = 1 + \sum_{k=1}^{K} V_k(\theta) p^k$$

**Issues and Treatments**:

Move: To mass operator?

Move: To pressure operator?

Move: To mass-flow operator?

Move: To friction nonlinearity?

Integrator: Leap-frogging?

# Ugly: Hyperbolicity

**Hyperbolic Part**:

$$\frac{\partial p}{\partial t} = -\frac{\partial q}{\partial x}$$
$$\frac{\partial q}{\partial t} = -c\frac{\partial p}{\partial x}$$

**Issues and Treatments**:

- Neglected kinetic term!
- Non-normal differential operator!
- Large reduced basis?
- Extra diffusion?
- Better IMEX?

# Ultimate Horror: Switchability

**Switched Elements**:

Valves: SISO & Two States

Compressors: MIMO & Multiple States

**Issues and Treatments**:

- Change graph topology and steady-state!
- Multiple valves and compressors!
- Global or "local" reduced order model?
- Prohibit switching during scenario?
- Igor & Victor: I am looking at you!

# Summary

The Good: System Structure, Friction Nonlinearity

The Bad: Non-Pipe Elements, Multi-Scale

The Ugly: Compressibility Nonlinearity, Hyperbolicity

**Ideas, Suggestions, War Stories?**

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