MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS MAGDEBURG

CSC COMPUTATIONAL METHODS IN SYSTEMS AND CONTROL THEORY

20 YEARS 1998-2018

Structured Cross-Covariance-Based Model Reduction Applied to Gas Network Models

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Computational Methods in Systems and Control
MPI Magdeburg

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Model Order Reduction for COupled Systems (MORCOS)
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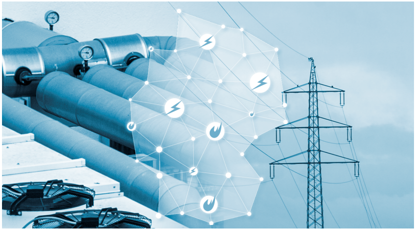
The Gas Network Challenge

Practically:

- Volatile Renewables.
- Natural Gas as Buffer.
- Fast Response.



Numerically:

- Coupled.
- Nonlinear.
- Hyperbolic.

Mathematical Key Technologies for Evolving Energy Grids

`http://mathenergy.de`

Project Partners:

- Fraunhofer SCAI
- Fraunhofer ITWM
- Humboldt Universität zu Berlin
- **Max Planck Institute Magdeburg** (Model Reduction)
- Technische Universität Berlin
- Technische Universität Dortmund
- PSI AG
- Universtät Trier

Funding:

- German Federal Ministry for Economic Affairs and Energy (BMWi)

MOR & COS

Model **O**rder **R**eduction:

- Large-Scale Networks
- Many-Query Setting

COupled **S**ystems:

- Pressure-Mass-Flux Coupling
- Interconnected System of Pipes

Act I

The Model

Pipe Model

Isothermal Euler Equations (**ISO-2**) for gas flow in a pipe[1]:

$$\begin{aligned} \frac{\partial}{\partial t}\rho &= -\frac{1}{S}\frac{\partial}{\partial x}q \\ \frac{\partial}{\partial t}q &= -S\frac{\partial}{\partial x}(R_S T z \rho) - Sg\rho\frac{\partial}{\partial x}h - \frac{\lambda}{2DS}\frac{q|q|}{\rho} \\ p &= R_S T_0 z \rho \end{aligned}$$

- Density: $\rho(x,t)$
- Mass-Flux: $q(x,t)$
- Pressure: $p(x,t)$
- Elevation: $h(x)$
- Constants: S, g, D
- Parameters: T_0, R_S
- Friction Factor: $\lambda(q)$
- Compressibility Factor: $z(p,T)$

[1] P. Benner, S. Grundel, C.H., C. Huck, T. Streubel, C. Tischendorf. **Gas Network Benchmark Models**. TRR154 Preprint, 2017.

Network Model

Network as graph $(\mathcal{N}, \mathcal{E})$:

- Node set $\mathcal{N}$ (Junctions)
- Edge set $\mathcal{E}$ (Pipes)

Node types:

- Internal nodes $\mathcal{N}_0$
- Supply nodes $\mathcal{N}_s$
- Demand nodes $\mathcal{N}_d$

Kirchhoff Rules[2]:

1. $p_i = p_s, \quad p_i \in \mathcal{N}_s$
2. $\sum_{j \in \mathcal{E}_-} q_j - \sum_{k \in \mathcal{E}_+} q_k = q_d, \quad q_d \in \mathcal{N}_d$

$\rightarrow$ Partial Differential Algebraic Equation (PDAE)

[2] T.P. Azevedo-Perdicoúlis and G. Jank. **Modelling Aspects of Describing a Gas Network through a DAE System**. 3rd IFAC Symposium on System Structure and Control 40(20): 40–45, 2007.

Spatial Discretization (PDAE $\to$ DAE):

- 1D Finite Difference
- Adaptive Refinement

Index Reduction (DAE $\to$ ODE):

1. Analytical[3]
2. Numerical[4]

[3] S. Grundel, L. Jansen, N. Hornung, T. Clees, C. Tischendorf, P. Benner. **Model Order Reduction of Differential Algebraic Equations Arising from the Simulation of Gas Transport Networks**. In: Progress in Differential-Algebraic Equations, Differential-Algebraic Equations Forum: 183–205, 2014.

[4] See: N. Banagaaya, S. Grundel, P. Benner. **Index-Aware MOR for Gas Transport Networks**. Friday: Session 12.

Boundaries & Quantities of Interest

Boundaries (Inputs)

- Pressure at supply nodes: p_s
- Mass-flux at demand nodes: q_d

Quantities of Interest (Outputs)

- Pressure at demand nodes: p_d
- Mass-flux at supply nodes: q_s

Input-Output System

Semi-Discrete Gas Network Model:

$$\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & A_{pq} \\ A_{qp} & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 0 & B_d \\ B_s & 0 \end{pmatrix} \begin{pmatrix} p_s \\ q_d \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p, q, p_s, q_d, \theta) \end{pmatrix}$$

$$\begin{pmatrix} p_d \\ q_s \end{pmatrix} = \begin{pmatrix} C_d & 0 \\ 0 & C_s \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

- Structured pressure and mass-flux components.
- Parameters: $\theta := \begin{pmatrix} T_0 & R_S \end{pmatrix}$.
- Parameter dependencies and compressibilities are lumped in f_q.
- Stiff linear part.
- Peculiar nonlinear part.

Reduction

Coupled, Nonlinear, Hyperbolic, Parametric Model:

$$\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & A_{pq} \\ A_{qp} & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 0 & B_d \\ B_s & 0 \end{pmatrix} \begin{pmatrix} p_s \\ q_d \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p, q, p_s, q_d, \theta) \end{pmatrix}$$

$$\begin{pmatrix} p_d \\ q_s \end{pmatrix} = \begin{pmatrix} C_d & 0 \\ 0 & C_s \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

- Square system (same number of inputs and outputs).
- Somewhat symmetric (linearization is Port-Hamiltonian[5]).
- (Non-Affine) parametrization.
- Distinct linear and nonlinear parts.
- "Unfortunately", we may not linearize.

[5] B. Liljegren-Sailer, N. Marheineke. **A Structure-Preserving Model Order Reduction Approach for Space-Discrete Gas Networks with Active Elements**. In: Progress in Industrial Mathematics at ECMI 2016: 439–446, 2018.

Reduced Quantities

Structured Projection-Based Reduced Order Model:

$$\begin{pmatrix}\dot{p}_r \\ \dot{q}_r\end{pmatrix} = \begin{pmatrix}0 & A_{pq,r} \\ A_{qp,r} & 0\end{pmatrix}\begin{pmatrix}p_r \\ q_r\end{pmatrix} + \begin{pmatrix}0 & B_{d,r} \\ B_{s,r} & 0\end{pmatrix}\begin{pmatrix}p_s \\ q_d\end{pmatrix} + \begin{pmatrix}0 \\ f_{q,r}(p_r, q_r, p_s, q_d, \theta)\end{pmatrix}$$

$$\begin{pmatrix}p_d \\ q_s\end{pmatrix} = \begin{pmatrix}C_{d,r} & 0 \\ 0 & C_{s,r}\end{pmatrix}\begin{pmatrix}p_r \\ q_r\end{pmatrix}$$

- Reducing truncated projections V_p, V_q.
- Reconstructing truncated projection: U_p, U_q.
- Reduced states: $p_r = V_p p$, $q_r = V_q q$.
- Reduced system matrices: $A_{pq,r} = V_p A_{pq} U_q, \quad A_{qp,r} = V_q A_{qp} U_p$.
- Reduced input matrices: $B_{d,r} = V_p B_d, \quad B_{s,r} = V_q B_s$.
- Reduced output matrices: $C_{d,r} = C_d U_p, \quad C_{s,r} = C_p U_q$.
- Reduced vector field: $f_{q,r}(p_r, q_r, p_s, q_d, \theta) = V_q f_q(U_p p_r, U_q q_r, p_s, q_d, \theta)$.

Reduction Rationale

Schematic Input-Output System:

$$\begin{pmatrix} p_s \\ q_d \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} p \\ q \end{pmatrix} \quad \mapsto \quad \begin{pmatrix} p_d \\ q_s \end{pmatrix}$$

$$\begin{array}{cc} \downarrow & \downarrow \\ \text{Input-to-State} & \text{State-to-Output} \\ \downarrow & \downarrow \\ \text{Controllability} & \text{Observability} \end{array}$$

$\rightarrow$ System Theoretic Approach: Balancing

Cross-Covariance

System Cross-Covariance:

$$X := \int_0^T F(t) G^*(t) \, \mathrm{d}t = \int_0^T \underbrace{x(t;\tilde{u})}_{F} \underbrace{[y^*(t;\tilde{x}_{0,1}), \dots, y^*(t;\tilde{x}_{0,N})]}_{G^*} \mathrm{d}t$$

- Assume SISO; for MIMO: Sum over SISOs[6].
- State-trajectory with perturbed steady-state input $x(t;\tilde{u})$.
- Output-trajectory with perturbed steady-state $y(t;\tilde{x}_{0,i})$.
- Purely data-driven computation using IMEX[7].
- Eigen-decomposition yields balancing transformation.

[6] C.H. and M. Ohlberger. **A note on the cross Gramian for non-symmetric systems**. System Science and Control Engineering 4(1): 199–208, 2016.

[7] S. Grundel, L. Jansen. **Efficient Simulation of Transient Gas Networks Using IMEX Integration Schemes and MOR Methods**. IEEE 54th Annual Conference on Decision and Control: 4579–4584, 2015.

Structured Cross-Covariance

Structured Cross-Covariance:

$$X_p = \int_0^T p(t;\tilde{u}) \, [y^*(t;\tilde{p}_{0,1}), \dots, y^*(t;\tilde{p}_{0,N_p})] \, \mathrm{d}t$$
$$X_q = \int_0^T q(t;\tilde{u}) \, [y^*(t;\tilde{q}_{0,1}), \dots, y^*(t;\tilde{q}_{0,N_q})] \, \mathrm{d}t$$

- Two individual cross-covariances for pressure and mass-flux.
- Both use all inputs $u = \begin{pmatrix} p_s & q_d \end{pmatrix}$ and outputs $y = \begin{pmatrix} p_d & q_s \end{pmatrix}$.
- This is a combination of:
  1. The empirical cross Gramian[8],
  2. with model reduction for interconnected systems[9].

[8] C.H. and M. Ohlberger. **Cross-Gramian-Based Combined State and Parameter Reduction for Large-Scale Control Systems**. Mathematical Problems in Engineering, 2014: 1–13, 2014.

[9] H. Sandberg and R.M. Murray. **Model reduction of interconnected linear systems**. Optimal Control Applications and Methods, 30(3): 225–245, 2009.

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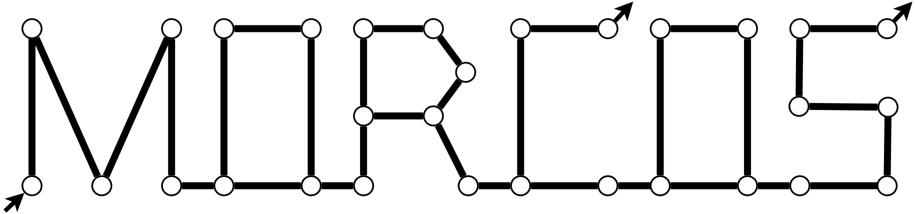
Experiments

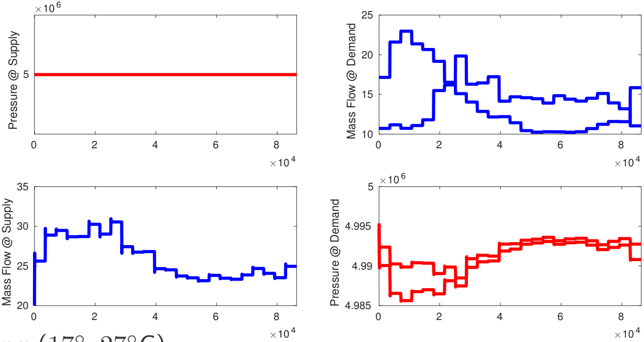


Pipe Network:

- 29 Junctions
- 32 Edges
- 3 Cycles
- 1 Supply node
- 2 Demand nodes

Input-Output System:

- 3 Inputs
- 637 States
- 3 Outputs



Offline Training ($17°$–$27°$C):

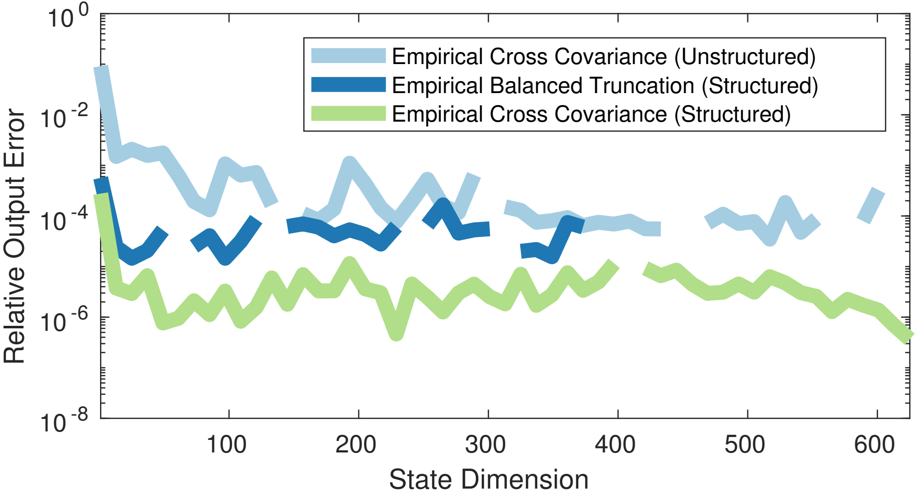
- Steady-state scenario
- Perturbations in p_s, q_d, p_0, q_0
- **1h** horizon, 1min resolution
- No data-driven reduction crime!

Standardized Load Profile[10] ($20°$–$25°$C):

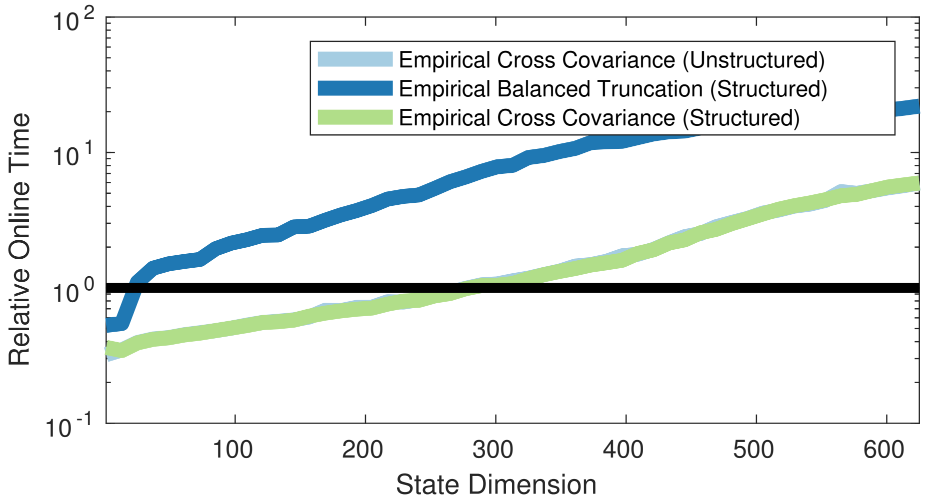
- Constant supply
- Demands: Single home & bakery
- **24h** horizon, 1min resolution
- on a Tuesday

[10] M. Hellwig. **Entwicklung und Anwendung parametrisierter Standard-Lastprofile**. PhD Thesis, TU München, 2003.

Relative L_2 Model Reduction Error



Relative Online Time



Research Software

`morgen` - **M**odel **O**rder **R**eduction for **G**as and **E**nergy **N**etworks

- Performs: Discretization, simulation, interfacing.
- Modular: Models, solvers, decouplers, reductors.
- In development[11].

`emgr` - **Em**pirical **Gr**amian Framework[12]

- Computes: Empirical cross covariances and empirical Gramians.
- Configurable: Inner product, solver and variants.
- Reductor backend to `morgen` using passed solver.

[11] by Simulation of Energy Systems (SES) team: S. Grundel, N. Banagaaya, Y. Qiu, C.H.

[12] C.H. `emgr` - **Empirical Gramian Framework** (Version **5.4**), 2018. `http://gramian.de`

Summary

Ingredients:

- Balancing controllability and observability.
- Empirical structured cross-covariance.
- Fast offline phase due to short training horizons.

Outlook:

- Error indicator is WIP.
- Compressors, valves and friends.
- Structured Gramian assembly.

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