MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

MORscore – Comparability of Model Reduction Algorithms

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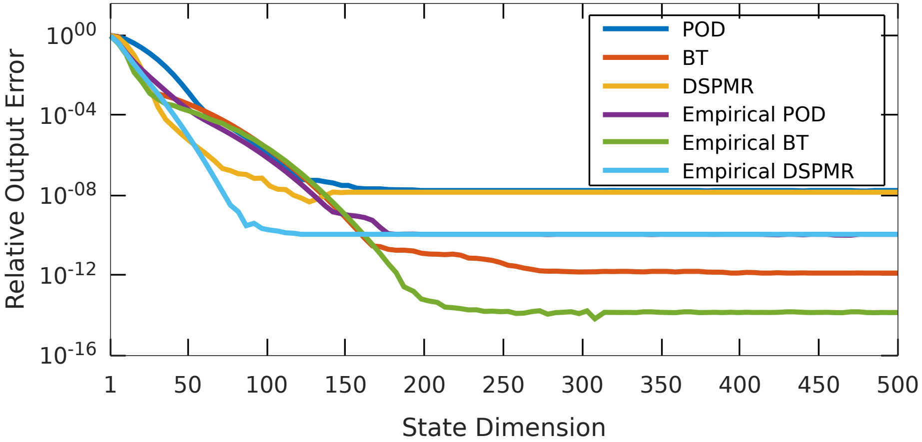
About

When we (should) compare MOR algorithms:

- New Methods (qualitative)
- MORwiki (quantitative)
- Software updates (regressions)
- Reviews (verification)
- Collaborations (recommendations)

But: What means better?



POD
BT
DSPMR
Empirical POD
Empirical BT
Empirical DSPMR
Relative Output Error
10^00
10^-04
10^-08
10^-12
10^-16
1
50
100
150
200
250
300
350
400
450
500
State Dimension

Setting

- How should we compare MOR algorithms?
  $\rightarrow$ It is a multi-objective problem.

- Can we assign a performance index to a method?
  $\rightarrow$ Like in computer benchmarking.

- Can we come up with a readable (ordered) score?
  $\rightarrow$ Humans should be able to easily comprehend.

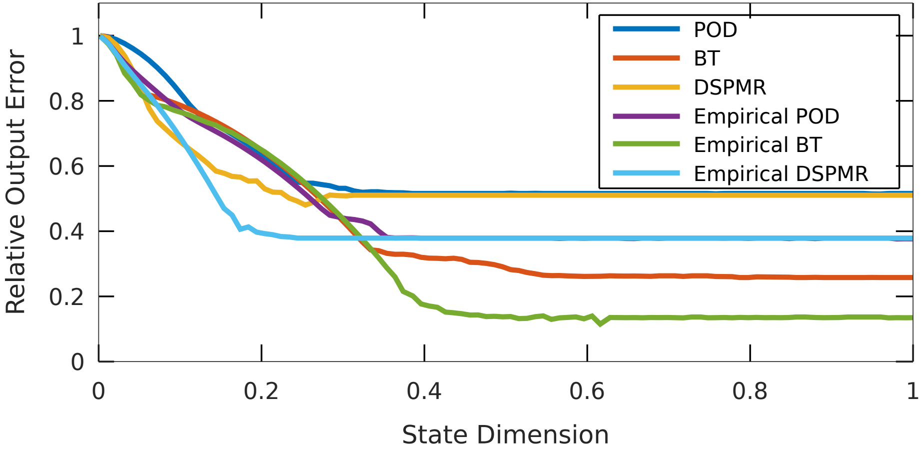
Preprocessing

1. Select maximum reduced order: $n_{\mathsf{max}} := 100$
2. Normalize orders: $1, 2, \dots, 100 \to \frac{1}{100}, \frac{2}{100}, \dots, \frac{100}{100}$
3. Normalize errors (via $\log_{10}$): $10^0, 10^{-1}, \dots, 10^{-16} \to 1, \frac{15}{16}, \dots, \frac{1}{16}$

Now we have:

- normalized orders $n_i \in (0, 1] \subset \mathbb{Q}$, and
- normalized errors $\varepsilon_i \in [0, 1] \subset \mathbb{R}$.

Normalized Example Plot



Error Map and Error Graph

Normalized Error Mapping:

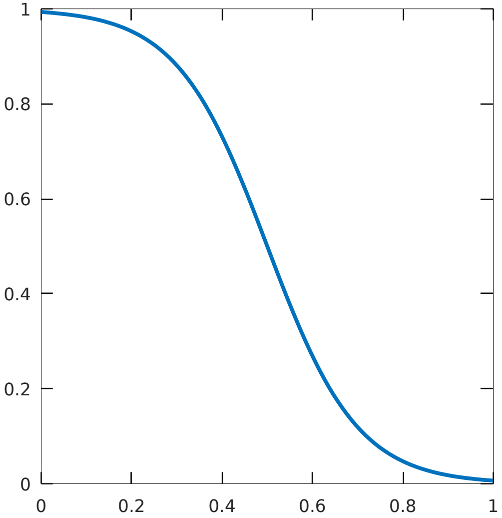
$$\varepsilon_{\text{MOR}} : (0, 1] \subset \mathbb{Q} \to [0, 1] \subset \mathbb{R}$$

Normalized Error Graph:

$$(n_i, \varepsilon_i)$$

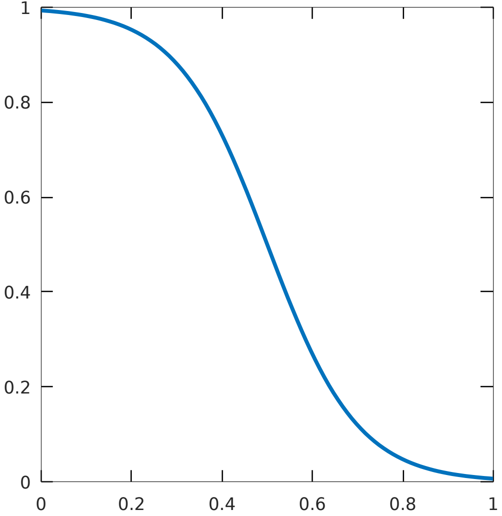
- Maps a normalized reduced order to a normalized relative log-error.
- Typically gathered through empirical testing.
- Alternatively error bounds or indicators could be used.

An Observation



- Errors start near one,
- decay with some speed,
- and then flatten out.

An Observation



- Errors start near one,
- decay with some speed,
- and then flatten out.

Like a sigmoid function!

A Sigmoid Hope

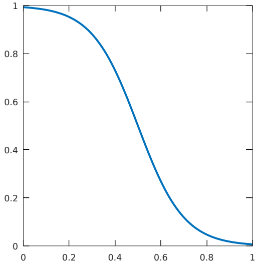
Sigmoid (Logistic Function):

$$s(x) = 1 - \frac{L}{1 + \mathrm{e}^{-k(x-x_0)}}$$

Parameters:

- L Minimum (higher means better)
- k Steepness (higher means better)
- x_0 Midpoint (higher means better)

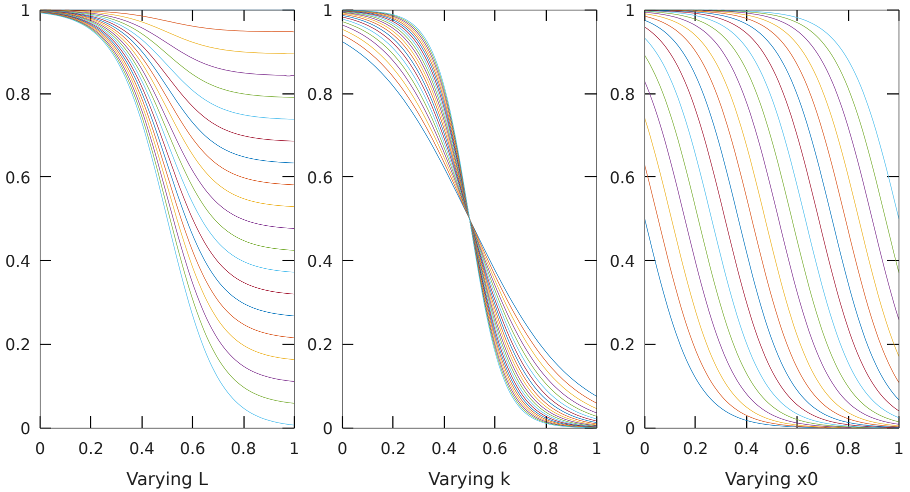
Score is least-squares fit to error graph.



Comments:

- How to compare two scores? In a norm?
- How should a norm be weighted?
- Is it useful?

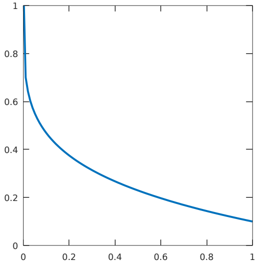
Sigmoid Family Portraits



The Monomial Strikes Back

Monomial:

$$m(x) = 1 - (x^d + c)$$



Parameters:

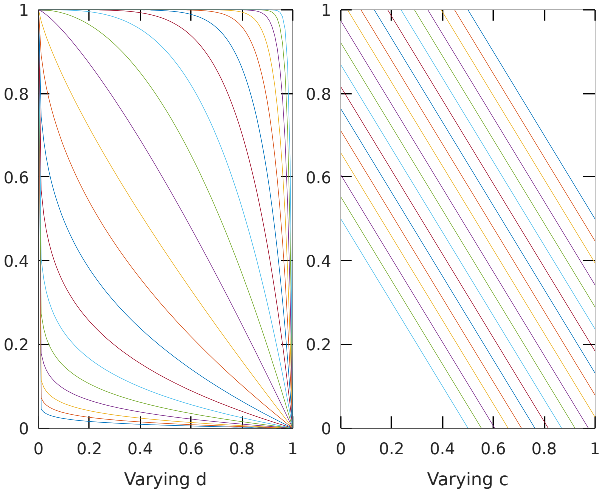
- $d > 0$ Degree (lower means better)
- c Shift (lower means better)

Score is least-squares fit to error graph.

Comments:

- How to compare two scores?
- What means a different shift?
- Is it useful?

Monomial Family Portraits



Return of the Trapezoid Knights

Trapezoid Rule:

$$A = 1 - \sum_{k=1}^{N} \frac{\Delta x}{2} y_{k-1} + y_k$$

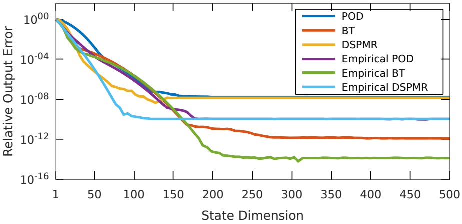
Candidate:

- $A =$ Area above curve

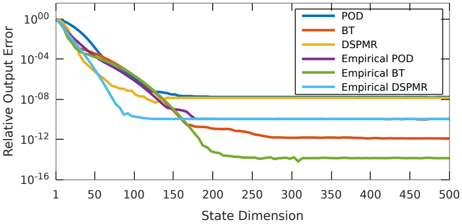
Comments:

- One score!
- Cheap computation.
- See also: [Curtis,Mitchell,Overton'17]

Comparison ($n_{\mathsf{max}} = 500$)



	Sigmoid (L, k, x_0)	Monomial (d, c)	Trapezoid
POD	$(0.48,\ 20.85,\ 0.14)$	$(0.15,\ -0.45)$	0.42
BT	$(0.74,\ 11.30,\ 0.20)$	$(0.33,\ -0.16)$	0.59
DSPMR	$(0.49,\ 30.32,\ 0.08)$	$(0.11,\ -0.45)$	0.45
Empirical POD	$(0.63,\ 13.83,\ 0.16)$	$(0.22,\ -0.30)$	0.53
Empirical BT	$(0.88,\ 10.70,\ 0.23)$	$(0.45,\ -0.01)$	0.68
Empirical DSPMR	$(0.62,\ 29.40,\ 0.10)$	$(0.16,\ -0.31)$	0.56



	Sigmoid (L, k, x_0)	Monomial (d, c)	Trapezoid
POD	$(0.48,\ 10.40,\ 0.27)$	$(0.24,\ -0.46)$	0.35
BT	$(0.75,\ 5.53,\ 0.40)$	$(0.47,\ -0.24)$	0.44
DSPMR	$(0.49,\ 15.15,\ 0.16)$	$(0.18,\ -0.44)$	0.41
Empirical POD	$(0.64,\ 6.51,\ 0.32)$	$(0.34,\ -0.32)$	0.43
Empirical BT	$(1.05,\ 4.18,\ 0.55)$	$(0.92,\ -0.03)$	0.49
Empirical DSPMR	$(0.62,\ 14.46,\ 0.20)$	$(0.27,\ -0.29)$	0.50

Summary

MORscore := The area fraction of the normalized log-error graph.

Questions:

- How to select maximum reduced order?
- Would this be useful to you?
- Should it be used in the MORwiki?
- How to handle unstable ROMs?
- Do you have other ideas for scores?