MAX PLANCK INSTITUTE
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CSC
COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

morgen

# Model Order Reduction for Gas and Energy Networks

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## `morgen`

- Modular,
- extensible,
- open
- test platform
- for: model order reduction
- of: multi-dimensional network dynamics,
- such as: **Gas**, Heat, Water;
- compatible with: MATLAB & OCTAVE;
- part of the MathEnergy software library.

# morgen Roadmap

- **0.9** Initial release (Paper submission: Q1/2020)
- **1.0** Stable release (After publication)
- **1.1** Update release (After extensions and testing)

# Affine Compressor Model

**Affine model (short “compressor pipe”):**

$$\dot{p}(t) = -p(t) + p_c(\theta)$$
$$\dot{q}(t) = 0$$

**See also:**

- Simple model[1]
- Factor model[2]
- Affine model[3]

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[1] K. Ehrhardt and M.C. Steinbach. **Nonlinear optimization in gas networks**. In Modeling, Simulation and Optimization of Complex Processes: 139–148. Springer, 2005.

[2] K. Sundar and A. Zlotnik. **State and parameter estimation for natural gas pipeline networks using transient state data**. IEEE Transactions on Control Systems Technology, 27(5): 2110–2124, 2019.

[3] T.P. Azevedo-Perdicoulis and G. Jank. **Modelling aspects of describing gas networks through a DAE system**. IFAC Proceedings Volume (3rd IFAC Symposium on Structure and Control), 40(20): 40–45, 2007.

# Model Order Reduction

**Data-Driven Model Reduction Algorithms:**

- Proper Orthogonal Decomposition
- Empirical Balanced Truncation
- Empirical Cross Gramian
- Empirical Non-Symmetric Cross Gramian
- Empirical Dominant Subspaces*
- Dynamic Mode Decomposition*

**Features:**

- Based on system theory.
- Galerkin projection.
- Error indidcator[4].
- Surprisingly good for hyperbolic systems[5].
- Works for nonlinear gas network models.

[4] P. Benner and C.H. **Cross-Gramian-Based Dominant Subspaces**. Advances in Computational Mathematics (Model Reduction of Parametrized Systems Topical Collection), 2019.

[5] S. Grundel, C.H. and J. Saak. **On Empirical System Gramians**. Proceedings in Applied Mathematics and Mechanics, 19: e201900006, 2019.

**Features:**

- Based on Koopman theory.
- Galerkin projection.
- Solve via generalized eigenvalue problem[6].
- Use dynamic mode decomposition with control[7].
- Robust for parametric gas network models.

[6] W. Zhang and M. Wei. **Solving generalized eigenvalue problem: an alternative approach for dynamic mode decomposition**. AIAA Scitech 2019 Forum: 1897, 2019.

[7] J.L. Proctor, S.L. Brunton, and J.N. Kutz. **Dynamic mode decomposition with control**. SIAM J. Applied Dynamical Systems, 15(1): 142–161, 2016.

# Hyperbolic Model Reduction I

**Hyperbolic Problem:**

$$\frac{\partial u_\mu}{\partial t} = -\frac{\partial f_\mu(u_\mu)}{\partial x}$$

- Slow decay of Kolmogorov $N$-width[8].
- Poor approximability in a linear space (i.e. via POD).
- $\rightarrow$ Try non-linear approximation spaces!

[8] C. Greif and K. Urban. **Decay of the Kolmogorov $N$-width for wave problems**. Applied Mathematics Letters, 96: 216–222, 2019.

# Hyperbolic Model Reduction II

**Nonlinear Approximation Spaces:**

- Split into transport and shape change operators[9] [10].
- Transport operator requires a non-linear approximation space.
- Shape change operator approximable by linear spaces.
- Not extendable to non-periodic boundaries!
- Residual minimization allows to drop periodicity assumption[11].
- WIP: hyper-reduction for residual minimization.
- Linearized Euler equation for pipelines already reducible.

[9] C.W. Rowley and J.E. Marsden. **Reconstruction equations and the Karhunen–Loeve expansion for systems with symmetry**. Physica D: Nonlinear Phenomena, 142(1–2): 1–19, 2000.

[10] S. Rave and M. Ohlberger. **Nonlinear reduced basis approximation of parameterized evolution equations via the method of freezing** Comptes Rendus Mathematique, 351(23–34): 901–906, 2013.

[11] K. Carlberg, D. Amsallem, P. Avery, M. Zahr and C. Farhat. **The GNAT nonlinear model reduction method and its application to fluid dynamics problems**. 6th AIAA Theoretical Fluid Mechanics Conference, 2011.

# Training Inputs and Excitation

**Training Input?**

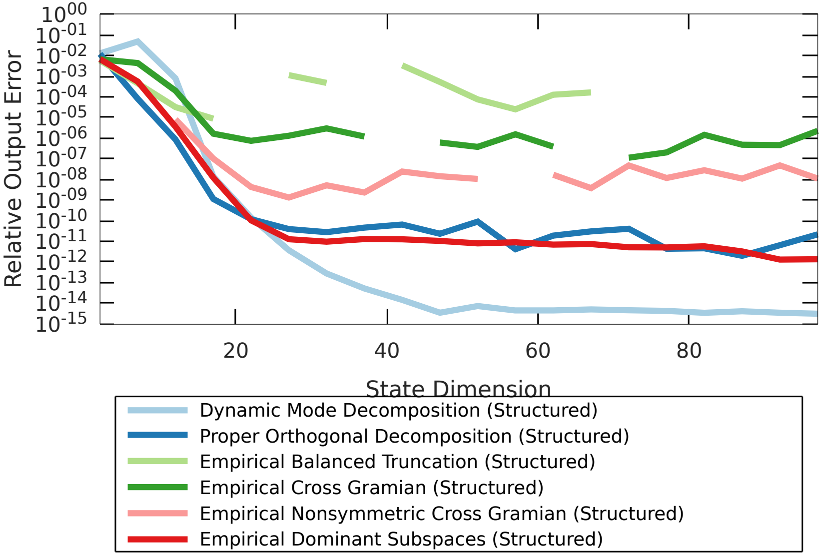
- Data-driven model reduction needs inputs.
- These training inputs cannot be scenario-specific,
- but need to excite the system suffciently.
- System identification investigates excitation[12].

**Excitation Comparison for a Pipeline:**

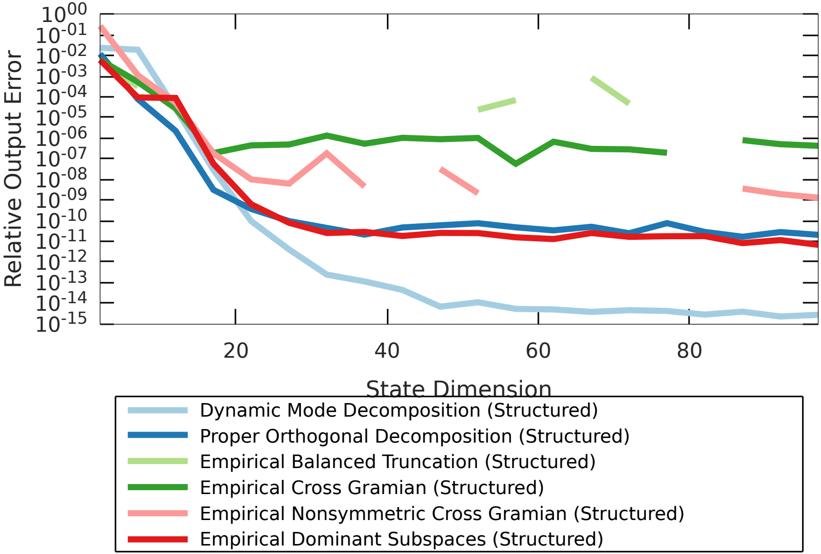
- Impulse
- Step
- Random Binary
- White Noise

[12] O. Nelles. **Nonlinear System Identification**. Springer, 2001.

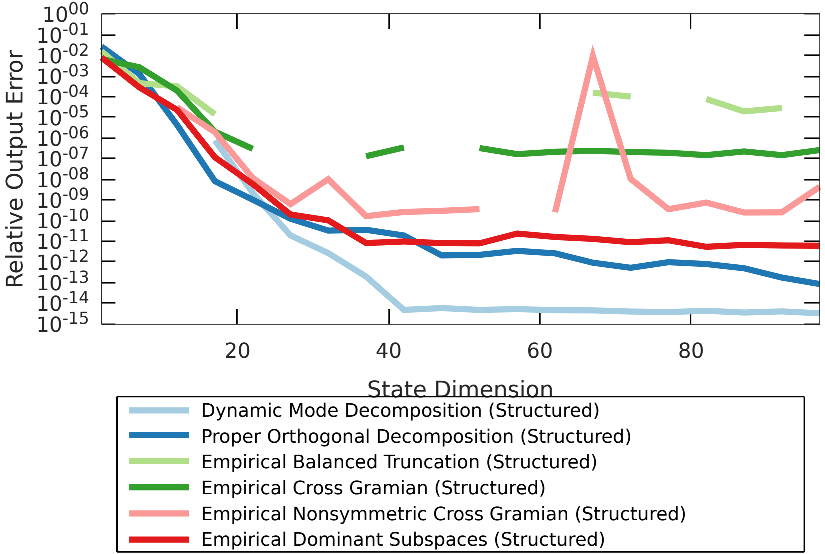
# Numerical Results (Impulse)



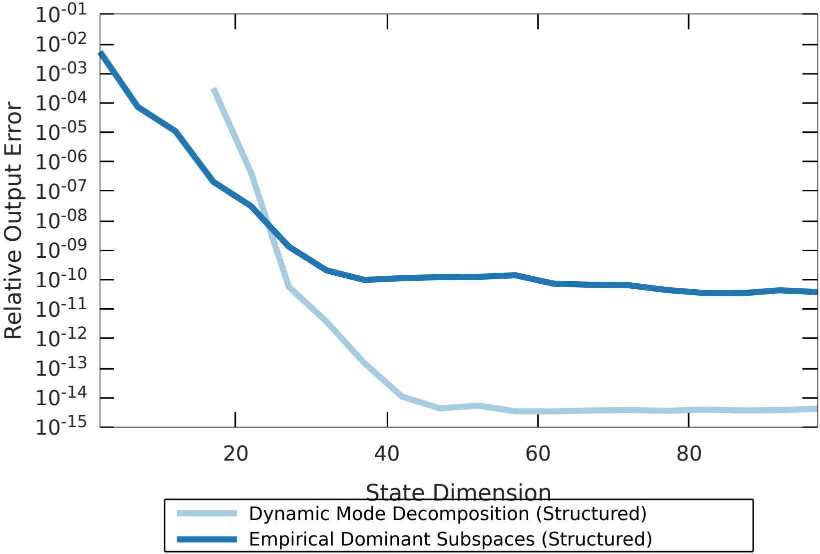
# Numerical Results (Step)



# Numerical Results (Random Binary)



# Numerical Results (White Noise)



# Notes

**Further Work:**

- CFL-based automatic refinement[13].
- IMEX solver evaluation.
- Sustainable scientific software[14] ("sequel" to RRR[15]).

[13] J.F. Helgaker, B. Müller and T. Ytrehus. **Transient Flow in Natural Gas Pipelines Using Implicit Finite Difference Schemes**. Journal of Offshore Mechanics and Arctic Engineering, 136(3): 031701, 2014.

[14] J. Fehr, C. Himpe, S. Rave and J. Saak. **Sustainable Research Software Hand-Over**. arXiv (Submitted), cs.GL: 1909.09469, 2019.

[15] J. Fehr, J. Heiland, C. Himpe and J. Saak. **Best Practices for Replicability, Reproducibility and Reusability of Computer-Based Experiments Exemplified by Model Reduction Software**. AIMS Mathematics (AIMS Math) 1(3): 261–281, 2016.

# Outlook

**Next:**

- Theory: Provably convergent hyper-reduction
- Networks: **SciGrid_Gas**
- Models: (nonlinear) Port-Hamiltonian
- Reductors: Empirical Balanced Gains
- Tests: Distributed integration testing
- Other: Visualization

https://himpe.science

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