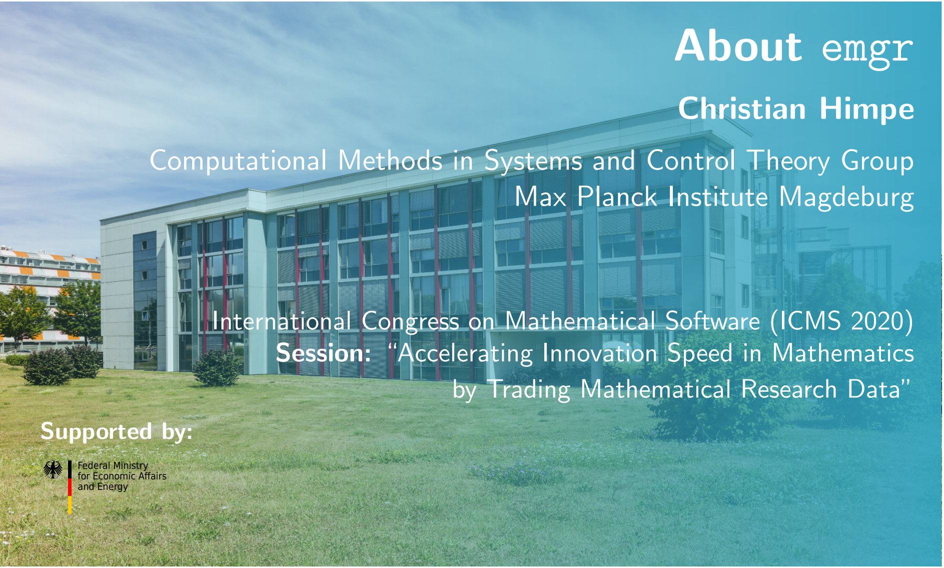
MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY



About emgr

Christian Himpe

Computational Methods in Systems and Control Theory Group
Max Planck Institute Magdeburg

International Congress on Mathematical Software (ICMS 2020)
Session: "Accelerating Innovation Speed in Mathematics by Trading Mathematical Research Data"

Supported by:

Federal Ministry
for Economic Affairs
and Energy

Outline

1. What is `emgr` (**EM**pirical **GR**amian Framework)?
2. Mathematical software as research data!

Locating emgr

- Applied mathematics

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  - **Mathematical system theory**
    - Modern system theory by Kalman (includes control theory)
    - Study of dynamical input-output systems

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    - Numerical linear algebra (matrix decompositions etc.)
    - Numerical analysis (differential equations etc.)

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  - **Computational science and engineering**
    - Modeling and discretization of engineering applications
    - Simulate, estimate, predict real-life behavior

Dynamical Input-Output System

General input-output system:

$$
\begin{aligned}
0 &= f(t, x(t), u(t), \theta, D^\alpha x(t)) \\
y(t) &= g(t, x(t), u(t), \theta)
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Example: **Gas network model**

$$\begin{pmatrix} E_p & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & A_{pq} \\ A_{qp} & 0 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 0 & B_d \\ B_s & 0 \end{pmatrix}\begin{pmatrix} s_p \\ d_q \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p, q, s_p) \end{pmatrix}$$
$$\begin{pmatrix} s_q \\ d_p \end{pmatrix} = \begin{pmatrix} 0 & C_q \\ C_p & 0 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$$

System Gramians

Core input-output system properties:

- Reachability (can the state be driven to any state by finite input?)
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Empirical Gramians

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- Use state trajectory data from (scaled) impulse responses.
- Impulse responses can also be computed for nonlinear systems!
- Observability can similarly be described by trajectory data.

Application:

Scoping `emgr`

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  - Combined state and parameter reduction (comred)

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Reduced order model:

$$\dot{x}_r(t) = (V(A(Ux_r(t)))) + (V(Bu(t)))$$
$$\tilde{y}(t) = \quad (C(Ux_r(t)))$$

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Empirical Gramians:

- Empirical Reachability Gramian
- Empirical Observability Gramian
- Empirical Linear Cross Gramian
- Empirical Cross Gramian
- Empirical Sensitivity Gramian
- Empirical Identifiability Gramian
- Empirical Joint Gramian

Initial release of emgr (0.9) at: MoRePaS 2 (2012)

Features:

- Open-source **Octave** and **MATLAB** toolbox, with **Python** variant.
- Interfaces for: Solver, inner product kernels & low-rank computation.
- Configurable, vectorized and parallelizable.

More info: https://gramian.de

C.H. emgr – **The Empirical Gramian Framework**. Algorithms 11(7): 91, 2018. doi:c9hj

Through the lens of `emgr`

Classes of Software

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- **Single-use software**
  - Computational proofs
  - Numerical experiments
  - Visualization generators

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Safety in numbers:

- Project website / personal website / institute website

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- Version (semantic version vs. calendar version)

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Semantic Enrichment

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Semantic Enrichment

ALL CAPS:

- README (plain text minimal documentation)
- RUNME (working example)
- CODE (code meta-data, i.e.: https://github.com/gramian/code-ini)

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Software Peer Review

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- Test review
  - Developer defines and describes a test,
  - based on an analytically known solution,
  - or a solution computed by alternative means.
  - Reviewer evaluates test quality and result.

Conservative values:

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  → document PDF & supplemental material `zip` archive

- **Optional** experimental technologies

  → Jupyter Notebooks, Code Ocean

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- Programming language longevity

  → FORTRAN, LISP, MATLAB

- Programming language readability

  → MATLAB, MAPLE

Summary

Do you want to know more?

- Empirical Gramians: `https://gramian.de`
- Best Practices: `https://doi.org/bsb2`
- Sustainable Software: `https://arxiv.org/abs/1909.09469`
- Mathematical Software: `https://doi.org/10.1016/C2013-0-11363-3`

`https://himpe.science`

Acknowledgment:
Supported by the German Federal Ministry for Economic Affairs and Energy, in the joint project: "**MathEnergy** – Mathematical Key Technologies for Evolving Energy Grids", sub-project: Model Order Reduction (Grant number: 0324019**B**).

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← Get slides

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