MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

Model Order Reduction for Gas and Energy Networks

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Model Reduction and Gas Networks

Isothermal Euler Equations on Graphs:

$$d_0 \, \partial_t p^* = D_p \partial_x q$$

$$\partial_t q^* = D_q \partial_x p - \left(D_q D_g \, d_0 \, p^* + D_f \, \frac{|q^*| \, q^*}{d_0 \, p^*} \right)$$

$$\mathcal{A}_0 q^* = \mathcal{B}_d \, d_q$$

A model reduction challenge:

- Parametric
- Nonlinear
- Hyperbolic
- Coupled
- Partial Differential(-Algebraic) Equation Systems

Isothermal Euler Equations on Graphs:

$$
\begin{aligned}
d_0\, \partial_t p^* &= D_p \partial_x q \\
\partial_t q^* &= D_q \partial_x p - \left(D_q D_g\, d_0\, p^* + D_f\, \frac{|q^*|\, q^*}{d_0\, p^*} \right) \\
\mathcal{A}_0 q^* &= \mathcal{B}_d\, d_q
\end{aligned}
$$

A model reduction challenge:

- Large-Scale
- Parametric
- Nonlinear
- Hyperbolic
- Coupled
- Partial Differential(-Algebraic) Equation Systems

$\rightarrow$ **M**odel **O**rder **R**eduction for **G**as and **E**nergy **N**etworks

Input-Output System:

$$\begin{pmatrix} E_p(\theta) & 0 \\ 0 & E_q \end{pmatrix}\begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & A_{pq} \\ A_{qp} & 0 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 0 & B_{pd} \\ B_{qs} & 0 \end{pmatrix}\begin{pmatrix} s_p \\ d_q \end{pmatrix} + \begin{pmatrix} 0 \\ F_c \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p, q, \theta) \end{pmatrix}$$

$$\begin{pmatrix} s_q \\ d_p \end{pmatrix} = \begin{pmatrix} 0 & C_{sq} \\ C_{dp} & 0 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$$

Test, compare, benchmark structured model reduction algorithms:

- Proper Orthogonal Decomposition (POD)
- Empirical Dominant Subspaces
- Empirical Balanced Truncation
- Balanced POD
- Empirical Balanced Gains
- Goal-Oriented POD
- Dynamic Mode Decomposition (DMD) Galerkin

Input-Output System:

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$$\begin{pmatrix} s_q \\ d_p \end{pmatrix} = \begin{pmatrix} 0 & C_{sq} \\ C_{dp} & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

Test, compare, benchmark structured model reduction algorithms:

- Proper Orthogonal Decomposition (POD)
- Empirical Dominant Subspaces *(first placed)*
- Empirical Balanced Truncation
- Balanced POD
- Empirical Balanced Gains
- Goal-Oriented POD
- Dynamic Mode Decomposition (DMD) Galerkin
- **Your** favorite model reduction method!

$\rightarrow$ https://doi.org/10.1186/s13362-021-00109-4