MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

CSC
COMPUTATIONAL METHODS IN
SYSTEMS AND CONTROL THEORY

Next-Gen Gas Network Simulation

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and Energy

About

Next-Gen	Modern numerics
Gas Network	Big infrastructure
Simulation	Digital twin

Why Next-Gen?

The Challenge:

- Renewables / Green energy
- Weather-dependend misery factors
- Natural gas / biogas / hydrogen mixing
- Gas / power coupling
- Volatility / uncertainty
- Daily day-ahead forecasting
- Minimizing consumption
- Virtual power plants

$\rightarrow$ **Many, many query setting**

What Means Next-Gen?

Simulation Stack:

- How to model?
- How to solve?
- How to accelerate solving?

Friction-Dominated Isothermal 1D Euler Equations:

$$
\begin{aligned}
\frac{1}{R_S\, T_0\, z_0}\, \partial_t p &= -\frac{1}{S}\, \partial_x q \\
\frac{1}{S}\, \partial_t q &= -\partial_x p - \Big(\frac{g\, \partial_x h}{R_S\, T_0\, z_0}\, p + \frac{R_S\, T_0\, z_0 \lambda_0}{2\, d\, S^2}\, \frac{|q|\, q}{p} \Big)
\end{aligned}
$$

$p(x,t)$	Pressure	g	Global gravity acceleration
$q(x,t)$	Mass-flux	T_0	Global mean temperature
$h(x)$	Pipe elevation	R_S	Global mean specific gas constant
d	Pipe diameter	z_0	Global mean compressibility factor
S	Pipe cross section	λ_0	Local friction factor

Gas Network Model

Conserved Quantities:

1. Kirchhoff $\rightarrow$ net mass-flux at every node is zero
2. Kirchhoff $\rightarrow$ sum of directed pressure drops in every loop is zero

Spatial Discretization:

- First order upwind finite differences
- Stable under CLF condition

Index Reduction:

1. Midpoint discretization
2. Endpoint discretization

Gas Transport Model

Compressors:

1. Multiplicative
2. Additive

Scales:

- Multi-scale due to coupling
- Change units from Pascal to Bar

Parameters:

1. Temperature
2. Specific gas constant

Input-Output System

Parametric, Structured, Nonlinear, Non-Normal, Square:

$$\overbrace{\begin{pmatrix} E_p(\theta) & 0 \\ 0 & E_q \end{pmatrix}}^{E} \begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \overbrace{\begin{pmatrix} 0 & A_{pq} \\ A_{qp} & 0 \end{pmatrix}}^{A} \begin{pmatrix} p \\ q \end{pmatrix} + \overbrace{\begin{pmatrix} 0 & B_{pd} \\ B_{qs} & 0 \end{pmatrix}}^{B} \begin{pmatrix} s_p \\ d_q \end{pmatrix} + \overbrace{\begin{pmatrix} 0 \\ F_c \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p, q, \theta) \end{pmatrix}}^{f}$$

$$\begin{pmatrix} s_q \\ d_p \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & C_{sq} \\ C_{dp} & 0 \end{pmatrix}}_{C} \begin{pmatrix} p \\ q \end{pmatrix}$$

Input:

- s_p Pressure @ supply
- d_q Mass-flux @ demand

State:

- p Pressure
- q Mass-flux

Output:

- s_q Mass-flux @ supply
- d_p Pressure @ demand

Port-Hamiltonian Structure

Input-State-Output Port-Hamiltonian System:

$$
\begin{aligned}
E\,\dot{x}(t) &= \overbrace{(J - R)\;\,Q}^{A}\,x(t) + \overbrace{(G - P)}^{B}\,u(t), \\
y(t) &= \underbrace{(G + P)^{\intercal} Q}_{C}\,x(t),
\end{aligned}
$$

- Mass Matrix: $E = E^{\intercal} > 0$
- Energy Flux: $J = -J^{\intercal}$
- Energy Dissipation: $R = R^{\intercal} > 0$
- Energy Storage: $Q = Q^{\intercal} > 0$
- Resistive Ports: P
- Control Ports: G

Port-Hamiltonian Duality

- Nonlinearity (friction) concentrates in dissipation R
- $R \to R(x)$, $\mathrm{im}(R(x))$ diagonal matrix, preserving sign
- Compressors disturb skew-symmetry of J

(Approximate) Adjoint Port-Hamiltonian System:

$$\begin{aligned} E\,\dot{x}(t) &= \overbrace{Q\big(-J-R\big)}^{A^\intercal}\,x(t) + \overbrace{Q(G+P)}^{C^\intercal}\,u(t), \\ y(t) &= \underbrace{(G-P)^\intercal}_{B^\intercal}\,x(t). \end{aligned}$$

Time Integration

Wishlist:

- Stiff linear part $\rightarrow$ Implicit solver!
- Nonlinear part $\rightarrow$ Explicit solver!
- External input $\rightarrow$ Fixed step solver!

$\rightarrow$ **Implicit-Explicit** Solver

Questions:

- Higher order $\rightarrow$ Passive methods?

Steady-State

Two-step steady state algorithm:

1a. Linear mass-flux steady-state: $A_{pq}\,\bar{q} = -B_{pd}\,\bar{d}_q$

1b. Linear pressure steady-state: $A_{qp}\,\bar{p} = -\Big(B_{qs}\,\bar{s}_p + F_c\Big)$

2. Corrected pressure steady-state: $A_{qp} p_{k+1} = -\Big(B_{qs}\bar{s}_p + F_c + f_q(p_k, \bar{q}, \theta)\Big)$

- Note, A and B do not depend on the parameter!
- Step 1a and Step 1b via QR-least-norm (in parallel).
- Repeat Step 2 until happy (reuse QR of Step 1b).
- Repeating Step 2 is a special case of an IMEX solver.
- If more accuracy is needed, iterate with 1st order IMEX solver.
- Practically, z_0 is also computed in Step 2.

Model Order Reduction

Wishlist:

- Input-output focus → System-theoretic methods!
- Handle nonlinearity → Data-driven methods!
- Handle hyperbolicity → Custom (but generic) training!
- Handle parametricity → Averaging trajectories!
- Preserve structure → Block-diagonal projectors!

→ (Structured) **Empirical System Gramians**

Questions:

- Square system → Cross Gramian?
- Preserve pH structure → Galerkin projector?
- Exploit pH structure → Observability as dual reachability?

Model Reduction Methods

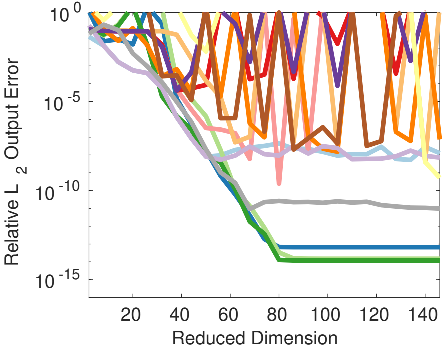
Tested Methods (implemented variants):

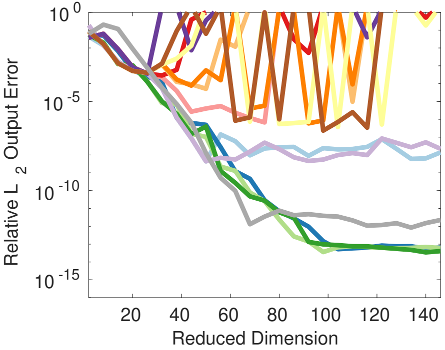
- **1** Structured Proper Orthogonal Decomposition
- **6** Structured Empirical Dominant Subspaces
- **2** Structured Balanced Proper Orthogonal Decomposition
- **6** Structured Empirical Balanced Truncation
- **1** Structured Goal-Oriented Proper Orthogonal Decomposition
- **6** Structured Empirical Balanced Gains
- **1** Structured Dynamic Mode Decomposition Galerkin

$\rightarrow$ **Short training, long testing**

Yamal-Europe Pipeline

- Structured Proper Orthogonal Decomposition (WR)
- Structured Empirical Dominant Subspaces (WR + WO)
- Structured Empirical Dominant Subspaces (WX)
- Structured Empirical Dominant Subspaces (WZ)
- Structured Balanced POD (WR + WO)
- Structured Empirical Balanced Truncation (WR + WO)
- Structured Empirical Balanced Truncation (WX)
- Structured Empirical Balanced Truncation (WZ)
- Structured Goal-Oriented POD (WR)
- Structured Empirical Balanced Gains (WR + WO)
- Structured Empirical Balanced Gains (WX)
- Structured Empirical Balanced Gains (WZ)
- Structured Dynamic Mode Decomposition Galerkin (WR)





Nonlinear midpoint discretization *vs* Linear endpoint discretization.

GasLib-134v2

Structured Proper Orthogonal Decomposition (WR)
Structured Empirical Dominant Subspaces (WR + WO)
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Structured Balanced POD (WR + WO)
Structured Empirical Balanced Truncation (WR + WO)
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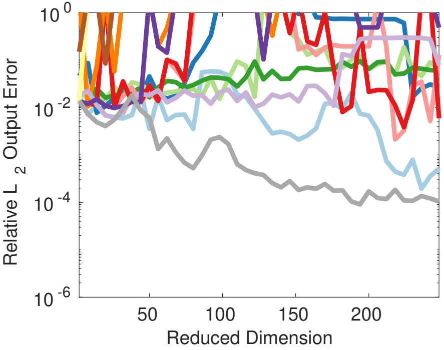
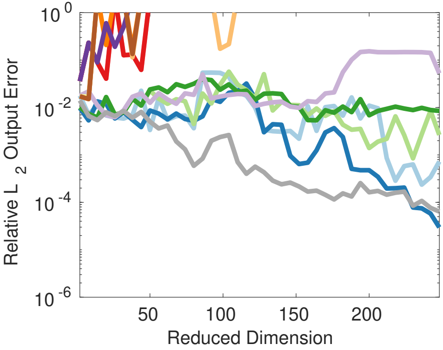
Structured Dynamic Mode Decomposition Galerkin (WR)

Numerical Results





Nonlinear midpoint discretization *vs* Linear endpoint discretization.

MORscore[1]

	ode_mid imex_1	ode_end imex_1	ode_end imex_1*
pod_r	0.14	0.14	
eds_ro	0.04	0.07	0.16
eds_wx	0.07	0.07	0.13
eds_wz	0.09	0.09	0.12
bpod_ro	0.05	0.07	0.00
ebt_ro	0.05	0.05	0.00
ebt_wx	0.00	0.00	0.00
ebt_wz	0.00	0.00	0.00
gopod_r	0.09	0.10	
ebg_ro	0.01	0.02	0.00
ebg_wx	0.00	0.00	0.00
ebg_wz	0.00	0.00	0.00
dmd_r	0.20	0.20	

[1] C. Himpe. **Comparing (empirical-Gramian-based) model order reduction algorithms**. In Model Reduction of Complex Dynamical Systems. Springer, 2021. `https://arxiv.org/abs/2002.12226`

Model **O**rder **R**eduction for **G**as and **E**nergy **N**etworks:

- Open-source (BSD-2-Clause)
- High-Level (MATLAB and OCTAVE)
- Efficient (Decomposition caching)
- Modular (Six modules)
- Configurable (Multi-level configuration)
- Extensible (Contributions welcome)

Modules (currently included):

2	Models	>**20**	Networks
4	Solvers	>**20**	Tests
23	Reductors	**6**	Tools

Next-Gen Gas Network Simulation

Model: **Port-Hamiltonian** (i.e. endpoint discretization)
Solver: **Implicit-Explicit** (i.e. 1st order IMEX)
Reductor: **Structured Galerkin** (i.e. dominant subspaces, DMD-Galerkin)

C. Himpe, S. Grundel, P. Benner. **Model Order Reduction for Gas and Energy Networks**. arXiv, 2021.
https://arxiv.org/abs/2011.12099

C. Himpe, S. Grundel. `morgen` – **Model Order Reduction for Gas and Energy Networks**. github, 2021.
https://git.io/morgen

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