WWU MÜNSTER

System Order Reduction for Gas and Energy Networks

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GAMM Annual Meeting (Section: Dynamics and Control)

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MM Mathematics Münster Cluster of Excellence

MaRDI

› MathEnergy Project

Model Reduction Sub-Project:

- Nonlinear Model Reduction for Gas Networks (no linearization)
- Input-Output (boundary actuators and sensors)
- Data-Driven ("unsupervised learning" of system-theoretic operators)
- Projection-Based
- Software Deliverable (!)

$\rightarrow$ **Platform for comparing model reduction for gas network simulation.**

C. Himpe, S. Grundel, P. Benner: **Model Order Reduction for Gas and Energy Networks**;
Journal of Mathematics in Industry 11: 13, 2021. doi:10.1186/s13362-021-00109-4

› About

MORGEN (**M**odel **O**rder **R**eduction for **G**as and **E**nergy **N**etworks)

- High volatility due to green energy transition.
- Gas-fired power plants can compensate.
- Weather-dependent consumption **and** production (Power2Gas).

$\rightarrow$ **Many simulations, every day, with a deadline.**

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SORGEN (**S**ystem **O**rder **R**eduction for **G**as and **E**nergy **N**etworks)

- Insufficient or sudden loss of supply.
- Ensuring supply of critical consumers.
- Increasing injection of biogas and hydrogen.

› Gas Network Modeling

Friction-Dominated 1D Isothermal Euler Equations:

$$\frac{1}{T_0\, R_S\, z_0}\,\partial_t p = -\frac{1}{S}\,\partial_x q$$

$$\partial_t q = -S\,\partial_x p - \Big(\overbrace{\frac{S\,g\,h_x}{T_0\,R_S\,z_0}}^{\text{Gravity}} p + \overbrace{\frac{T_0\,R_S\,z_0 \lambda_0}{2\,d\,S}\,\frac{|q|\,q}{p}}^{\text{Friction}} \Big)$$

1. Conservative interconnection (Kirchhoff laws)
2. Index reduction (Analytic)
3. Refinement strategy (CFL condition)
4. Spatial discretization (Finite differences)

Square Input-Output-System:

$$\begin{pmatrix} E_p(\theta) & 0 \\ 0 & E_q \end{pmatrix} \begin{pmatrix} \dot{p} \\ \dot{q} \end{pmatrix} = \begin{pmatrix} 0 & A_{pq} \\ \hat{A}_{qp} & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} 0 & B_{pd} \\ B_{qs} & 0 \end{pmatrix} \begin{pmatrix} s_p \\ d_q \end{pmatrix} + \begin{pmatrix} 0 \\ f_q(p, q, \theta) \end{pmatrix}$$

$$\begin{pmatrix} s_q \\ d_p \end{pmatrix} = \begin{pmatrix} 0 & C_{sq} \\ C_{dp} & 0 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix}$$

› Spatial Discretizations

Identifier	Name	Index
`ode_mid`	Midpoint Discretization	0
`ode_end`	Endpoint Discretization	0

S. Grundel, N. Hornung, B. Klaassen, P. Benner, T. Clees: **Computing surrogates for gas network simulation using model order reduction**; in: Surrogate-based modeling and optimization: 189–212, 2013. 10.1007/978-1-4614-7551-4_9
S. Roggendorf: **Model order reduction for linearized systems arising from the simulation of gas transportation networks**; Master's thesis. University of Bonn; 2015.

› Spatial Discretization

Endpoint Discretization:

- Non-Normal (hyperbolic PDE model)
- Structured (two coupled variables)
- Nonlinear (friction, compressor, etc.)
- Parametric (temperature, specific gas constant)
- **Port-Hamiltonian** (allows approximate adjoint)

$\rightarrow$ **Interesting** (challenging) **model reduction setting!**

› Temporal Discretizations

Identifier	Name	Order
`imex1`	Implicit-Explicit	1
`imex2`	Implicit-Explicit-Runge-Kutta	2
`cnab2`	Crank-Nicolson-Adams-Bashforth	2
`ode23s`	Rosenbrock	2
`rk2hyp`	Hyperbolic Runge-Kutta	2
`rk4hyp`	Hyperbolic Runge-Kutta	4
`rk4`	Classic Runge-Kutta	4

› Temporal Discretization

First Order Implicit Explicit:

- Explicit Euler combined with implicit Euler.
- Not a Runge-Kutta method.
- One nonlinear vector field evaluation.
- One linear problem (minimum for every solver).
- Matches spatial discretization order.

$\rightarrow$ **Most efficient** (among tested) **time stepper!**

› Reductors

Identifier	Name	Type
`pod`	Proper Orthogonal Decomposition (POD)	Galerkin
`gopod`	Goal-Oriented POD	Galerkin
`dspmr`	Dominant Subspace Projection Model Reduction	Galerkin
`mpod`	Modified POD	Oblique
`bpod`	Balanced POD	Petrov-Galerkin
`bt`	Balanced Truncation	Petrov-Galerkin
`bg`	Balanced Gains	Petrov-Galerkin
`dmd`	Dynamic Mode Decomposition Galerkin	Galerkin

› Reductor

General Findings:

- Galerkin methods much more stable than Petrov-Galerkin.
- Structure preserving (block-diagonal) projections necessary.
- Step input training for data-driven Gramian computation.
- Port-Hamiltonian property enables fast ROM computation.

Dominant Subspace Projection Model Reduction:

- Conjoins most reachable and most observable subspaces.
- Stability-preserving for considered (linearized) models.
- $\mathcal{H}_2$ error-bound and *a-priori* $\mathcal{H}_2$ error-indicator.

$\rightarrow$ **Data-driven projection-based model reduction works here!**

› Software

MORGEN (**M**odel **O**rder **R**eduction for **G**as and **E**nergy **N**etworks)

- **For engineers:** Test model reduction on your networks.
- **For mathematicians:** Test your model reduction on gas nets.
- **MATLAB & Octave** compatible
- **Open source** licensed
- **Version 1.2** upcoming

`https://git.io/morgen`

› Experimental Design

- Generic training input (step)
- Designed or random test scenarios
- Short training periods (1–12h)
- Long testing periods (24h)
- Generic training parameter sampling (sparse grid)
- Random test parameter sampling (uniform distributed)

Evaluation:

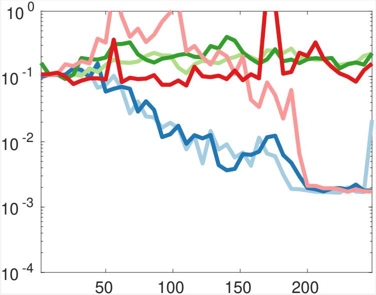
- Relative $L_2 \otimes L_2$ model reduction output error
- **MORscore** benchmark scoring

› Chinese Transport Network

- ≈8600km Total length
- 5 Inflow nodes
- 3 Outflow nodes
- 38 Compressor stations

From:
L. Jin, A.K. Wojtanowicz: **Optimization of Large Gas Pipeline Network – A Case Study in China**; Journal of Canadian Petroleum Technology 49(4): 36–43, 2010.
L. Jin, A.K. Wojtanowicz: **Optimization of Large Gas Pipeline Network in China – a Feasibility Study**; Canadian International Petroleum Conference: 174, 2008.

› Numerical Results



MORscores:	$\mu \in [0, 1)$
POD (WR)	0.12
DSPMR (WR + WR*)	0.12
DSPMR (WX*)	0.05
DSPMR (WZ*)	0.04
GOPOD (WR)	0.07
DMD Galerkin (WR)	0.06

› Summary

Recommended Ensemble:

- Model: **pH** (port-Hamiltonian Endpoint Discretization)
- Application: **Gas** (Gas Transport Pipeline Network)
- Solver: **IMEX** (1st Order Implicit Explicit Time Stepper)
- Reductor: **DSPMR** (Dominant Subspace Projection Model Reduction)
- Software: **MORGEN** (Model Order Reduction for Gas and Energy Networks)

`https://doi.org/gndbmv` (Paper)

`https://git.io/morgen` (Code)

`https://himpe.science` (Slides)